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# HYBRID STRUCTURES WITH APPLICATIONS FROM THE VIEWPOINT OF NEUTROSOPHIC MODEL

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**ABSTRACT.** In this work, we have presented the concept of "neutrosophic fuzzy bitopological ideal" (NFBI), which is a generalization of neutrosophic fuzzy ideal (NFI) that can deal with two topologies on a set. NFBI can capture the uncertainty, indeterminacy, and inconsistency of the security requirements and preferences of the entities in these applications. We had developed a neutrosophic fuzzy bitopological local function (NFBF), which is a function that assigns a NFS to each entity based on its local information, such as its location, speed, direction, trust level, and security level. NFBLF can reflect the dynamic and heterogeneous characteristics of the entities and their preferences. We had proposed a neutrosophic fuzzy bitopological ideal security scheme (NFBISS), which is a protocol that uses NFBI and NFS to distribute session keys between entities in these applications. NFBISS can adapt to the changing network conditions and select the optimal key length and encryption algorithm for each session based on NFBLF..

**KEYWORDS:** *MANET; Security; Wireless; Neutrosophic fuzzy ideal bitopology; PKI; KNN.*

## 1. INTRODUCTION

The essential concepts of neutrosophic fuzzy set (NBIS) were introduced by Smarandache in [3,4]. Also Salama presented the neutrosophic topological spaces, neutrosophic crisp sets and many applications in computer science and information system [6,8,9-13], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. This dissertation aims to introduce and investigate some new neutrosophic notions via neutrosophic ideals. Neutrosophy serves as a foundation for new mathematical theories that generalize classical and fuzzy concepts [1,2,5], with neutrosophic set theory being an example. The paper you are referring to aims to introduce and investigate novel neutrosophic notions using neutrosophic ideals.

We, also generalized the notion of neutrosophic pairwise open sets, neutrosophic pairwise closed sets, neutrosophic pairwise continuity and neutrosophic pairwise open functions due to Abd El-Monsef et al. [1].

Salama et al. [10,11] had introduced and studied the concept of neutrosophic bitopological spaces. Bitopological spaces considered as a generalization of topological spaces that have two distinct topologies on the same set. Neutrosophic bitopological spaces are a way of extending the notion of neutrosophic topological spaces to bitopological spaces. Some possible applications of neutrosophic bitopological spaces are in computer science and information systems.

Some papers [10,13] discuss similar ideas, such as:

- Security in MANET based on PKI using fuzzy function, which proposes a fuzzy model to decide the key length for the current session and a key distribution scheme between nodes in MANET.
- A neutrosophic theory based security approach for fog and mobile-edge computing, which uses neutrosophic set theory to select optimal security services according to the requirements of mobile users in the fog and mobile-edge computing environment.

- In [13-17], the first author presented some different applications of topological ideal and fuzzy topological ideal in different areas.
- "Neutrosophic fuzzy set" and its applications in decision making , which introduces the notion of neutrosophic fuzzy set (NFS) by combining fuzzy set with neutrosophic set, and proposes some distance and similarity measures between NFSs for decision making problems.

The research that specifically mentions neutrosophic fuzzy data bitopology security this might be a novel idea that you can explore further. Neutrosophic set is an extension of fuzzy and intuitionistic fuzzy set that can handle inconsistent, indeterminate and incomplete information using truth, indeterminacy and falsity membership grades. Neutrosophic bitopology is an extension of fuzzy topology that can deal with neutrosophic sets. Perhaps you can use these concepts to design a new security model for MANET using PKI and NFS.

## 2. TERMNOLOGIES

Theoughout this work, the following mathematical symploes are used

| Sympole | Phisical meaning |
|---|---|
| **NBIS** | Neutrosophic bitoplogical ideal space |
| **PKI** | Public key infrstrucure |
| **NFS** | Neutrosophic fuzzy set |
| **NFI** | Neutrosophic fuzy ideal |
| **NFBI** | Neutrosophic fuzzy bitoplogical ideal |
| **NFTS** | Neutrosophic fuzzy topological space |
| **NFBS** | Neutrosophic fuzzy bitoplogical spcae |

The axioms of openness and closeness for a neutrosophic fuzzy bitopological space are similar to those for a neutrosophic fuzzy topological space, but they apply to both topologies. For example, a collection T of neutrosophic fuzzy sets on a non-empty set X is called a neutrosophic fuzzy topology on X if it satisfies the following conditions[1]:

- The empty set and the universe set X are in T.
- The union of any family of sets in T is in T.
- The intersection of any finite family of sets in T is in T.

A pair (X, T) where X is a non-empty set and T is a collection of a neutrosophic fuzzy sets on X is called a NFTS. A neutrosophic fuzzy set A on X is called neutrosophic fuzzy open (respectively, neutrosophic fuzzy closed) if A belongs to T (respectively, its complement belongs to T).

A pair (X, $T_1$, $T_2$) where X is a non-empty set and $T_1$ and $T_2$ are two neutrosophic fuzzy topologies on X is called a NFBS. A pair of neutrosophic fuzzy sets (A, B) on X is called neutrosophic fuzzy open (respectively, neutrosophic fuzzy closed) with respect to the first (respectively, second) topology if A (respectively, B) is neutrosophic fuzzy open (respectively, neutrosophic fuzzy closed) in (X, $T_1$) (respectively, (X, $T_2$)).

A pair of neutrosophic fuzzy sets A and B in a neutrosophic fuzzy topological space (X, T) is called a neutrosophic fuzzy pairwise open set if for every x in X, there exists a neutrosophic fuzzy open set U such that $A(x) \leq U(x)$ and $B(x) \leq U(x)$. This means that the degrees of membership and indeterminacy of x in both A and B are less than or equal to the degrees of membership and indeterminacy of x in some neutrosophic fuzzy open set U. Neutrosophic fuzzy pairwise open sets can be used to study the properties of neutrosophic fuzzy bitopological spaces. For every notations and terminology not defined here, we follow [12,17].

## 3. SOME CONSIDERED EXAMPLES OF NEUTROSOPHIC FUZZY BITOPOLOGICAL SPACES

**EXAMPLE 3.1.** The pair (X, $T_1$, $T_2$), where X is any non-empty set and $T_1$ and $T_2$ are the discrete and indiscrete "neutrosophic fuzzy topologies" on X, respectively. This is a trivial example of a "neutrosophic fuzzy bitopological space", where every pair of neutrosophic fuzzy sets is neutrosophic fuzzy open and neutrosophic fuzzy closed with respect to both topologies[1].

- The pair (W, $T_1$, $T_2$), where W = {$k_1$, $k_2$, $k_3$} and $T_1$ and $T_2$ are the following collections of neutrosophic fuzzy sets on W[2]:
  $T_1 = \{< k_1, 0.5, 0.5, 0 >, < k_2, 0.5, 0.5, 0 >, < k_3, 0.5, 0.5, 0 >, < k_1, k_2, 0.5, 0.5, 0 >, < k_1, k_3, 0.5, 0.5, 0 >, < k_2, k_3, 0.5, 0.5, 0 >, < k_1,k_2,k_3 >\}$,
  $T_2 = \{< k_1 >, < k_2 >, < k_3 >, < k_1,k_2 >, < k_1,k_3 >, < k_2,k_3 >, < k_1,k_2,k_3 >\}$.
  This is an example of a neutrosophic fuzzy bitopological space that is not a neutrosophic fuzzy topological space.

  The pair (X, $T_1$, $T_2$), where X is any nonempty set and T1 and T2 are two "neutrosophic fuzzy topologies" on X that are generated by two different neutrosophic fuzzy bases B1 and B2 on X. This is an example of a neutrosophic fuzzy bitopological space that can be constructed from any two neutrosophic fuzzy bases on the same set.

**DEFINITION 3.1.** [7,11,12] "A neutrosophic fuzzy ideal bitopological space" is a generalization of a "neutrosophic fuzzy bitopological space" with an additional structure of a neutrosophic fuzzy ideal. A neutrosophic fuzzy ideal on a non-empty set X is a neutrosophic fuzzy set L on X such that for any x, y in X and any r in [0, 1], the following conditions hold[1]:

- $L(x) \geq r$ implies $L(x) = 1$.
- $L(x) \leq r$ implies $L(x) = 0$.
- $L(x) + L(y) \leq L(x \vee y)$.

- $L(x) + L(y) \geq L(x \wedge y)$.

A pair (X, L) where X is a nonempty set and L is a neutrosophic fuzzy ideal on X is called a neutrosophic fuzzy ideal space A neutrosophic fuzzy set A on X is called L-open (respectively, L-closed) if A belongs to L (respectively, its complement belongs to L).

A quadruple (X, $T_1$, $T_2$, L) where (X, $T_1$, $T_2$) is "a neutrosophic fuzzy bitopological space" and (X, L) is a neutrosophic fuzzy ideal space is called "a neutrosophic fuzzy ideal bitopological space". A pair of neutrosophic fuzzy sets (A, B) on X is called L-open (respectively, L-closed ) with respect to the first (respectively, second) topology if A (respectively, B) is L-open (respectively, L-closed) in (X, T1) (respectively, (X, T2)).

**DEFINITION 3.2**: As L(x)=1 for all x∈X, L is the neutrosophic fuzzy ideal on X. Accordingly, A belongs to the ideal L for any neutrosophic fuzzy set A on X if its "value" at each point x∈ X (in terms of truth, indeterminacy, and falsehood) is less than or equal to the corresponding "value" of L(x). Every neutrosophic fuzzy set on X in this particular instance belongs to the neutrosophic fuzzy ideal L since L(x)=1 for every x.

**EXAMPLE 3.2.** The quadruple (W, $T_1$, $T_2$, L), where W = {$k_1$, $k_2$, $k_3$} and $T_1$ and $T_2$ are the following collections of neutrosophic fuzzy sets on W:

$T_1$ = {< $k_1$, 0.5, 0.5, 0 >, < $k_2$, 0.5, 0.5, 0 >, < $k_3$, 0.5, 0.5, 0 >, < $k_1$, $k_2$, 0.5, 0.5, 0 >, < $k_1$, $k_3$, 0.5, 0.5, 0 >, < $k_2$, $k_3$, 0.5, 0.5, 0 >, < $k_1$,$k_2$,$k_3$ >},

$T_2$ = {< $k_1$ >, < $k_2$ >, < $k_3$ >, < $k_1$,$k_2$ >, < $k_1$,$k_3$ >, < $k_2$,$k_3$ >, < $k_1$,$k_2$,$k_3$ >},

L = {< $k_1$ >, < $k_2$ >, < $k_3$ >, < $k_1$,$k_2$,$k_3$ >}.

This is an example of "a neutrosophic fuzzy ideal bitopological space" that is not "a neutrosophic fuzzy bitopological space".

- The quadruple (X, $T_1$, $T_2$, L), where X is any nonempty set and $T_1$ and $T_2$ are two "neutrosophic fuzzy topologies" on X that are generated by two different "neutrosophic fuzzy" bases $B_1$ and $B_2$ on X, and L is any neutrosophic fuzzy ideal on X that contains both $B_1$ and $B_2$. This is an example of a neutrosophic fuzzy ideal bitopological space that can be constructed from any two neutrosophic fuzzy bases and any neutrosophic fuzzy L- on the same set.

**DEFİNİTİON 3.3**. [7] "A neutrosophic fuzzy pairwise local function is a function that maps a neutrosophic fuzzy bitopological space to another neutrosophic fuzzy bitopological space and preserves some properties of neutrosophic fuzzy pairwise L-open sets. It is defined as follows":

"Let (X, $T_1$, $T_2$) and (Y, $S_1$, $S_2$) be two neutrosophic fuzzy bitopological spaces and f: X → Y be a function. Then f is called a neutrosophic fuzzy pairwise local function if for each x in X and each neutrosophic fuzzy pairwise open set (V, W) in (Y, $S_1$, $S_2$) containing f(x), there exists a neutrosophic fuzzy pairwise open set (U, Z) in (X, $T_1$, $T_2$) containing x such that f(U) ⊆ V and f(Z) ⊆ W".

## 4. On Neutrosophic Fuzzy Pairwise L-Local Functions.

Salama ,et al. in [11] introduced each of "neutrosophic fuzzy pairwise L- open sets" and "neutrosophic fuzzy pairwise" L-closed sets and some new concepts that generalize the notions of "neutrosophic fuzzy pairwise" open sets and "neutrosophic fuzzy pairwise" closed sets via neutrosophic fuzzy ideals. They are defined as follows:

- A subset A of a neutrosophic topological space (W, τ, L) is called a "neutrosophic fuzzy pairwise ideal open set" if for each x ∈ A, there exists a neutrosophic fuzzy ideal L such that x ∈ L ⊆ A. A subset A of a neutrosophic topological space (X, τ, L) is called a neutrosophic fuzzy pairwise ideal closed set if its complement X \ A is a neutrosophic fuzzy pairwise ideal open set. "Neutrosophic fuzzy pairwise" local functions are generalizations of "neutrosophic fuzzy" local functions and neutrosophic fuzzy pairwise continuous functions. They can be used to study the properties and relations of neutrosophic fuzzy bitopological spaces and their mappings.

**DEFİNİTİON 4.1.** [11,12] "Given $(X,\tau_i,L)$. i∈{1.2} be a nflbts with neutrosophic ideal L on X , $\mu.\sigma.u \in I^X$. Then < μ,σ,u > is said to be :

(i) Neutrosophic L pairwise $\tau^*_i$-closed,i∈{1,2} may be have two types
Type 1: if $\mu^* \leq \mu, u^* \geq u, \sigma^* \leq \sigma$ .
Type 2: if $\mu^* \leq \mu, u^* \geq u, \sigma^* \geq \sigma$ (or PLN-$^*$closed) if < μ,σ,u > $^*\leq$ < μ,σ,u >

(ii) Fuzzy neutrosophic pairwise NLPL-dense – in – itself may be have two types
Type 1: if $\mu^* \leq \mu, u^* \geq u, \sigma^* \leq \sigma$ .
Type 2: if $\mu^* \leq \mu, u^* \geq u, \sigma^* \geq \sigma$ (or PLN$^*$-dense-in – itself ) if < μ,σ,u > ⊆ < μ,σ,u >$^*$.

(iii) Neutrosophic pairwise L$^*$-perfect if < μ,σ,u > is PLN$^*$-closed and PLN$^*$- dense – in itself".

**THEOREM 4.1:** Given $(X,\tau_i.L)$. i∈{1.2} be a nflbts with "neutrosophic ideal" L on X , $\mu.\sigma.u \in I^X$ then <μ,σ,u> is:

(i) PLN$^*$- closed iff NLcl$^*$(< μ,σ,u > ) =< μ,σ,u > .
(ii) PLN$^*$- dense – in – itself iff NLcl$^*$(< μ,σ,u > )= < μ,σ,u > $^*$.
(iii) PLN$^*$- perfect iff NLcl$^*$(< μ,σ,u > )= < μ,σ,u >$^*$=< μ,σ,u > .

**PROOF:** Follows directly from the neutrosophic pairwise closure operator Ncl$^*$ for a neutrosophic pairwise $\tau^*_i$(L). i ∈ {1.2} and Definition 3.1.

**REMARK 4.1:** One can deduce that

(i) Every PLN*-dense- in – itself is "a neutrosophic pairwise" dense set.

(ii) Every neutrosophic pairwise ideal closed (resp. "neutrosophic pairwise ideal" open) set is PLN*-closed (resp. $PLN\tau^{*}_{i}$ − open. $i \in \{1.2\}$),

**COROLLARY 4.1:** Given $(X,\tau_{i.}L).i\epsilon\{1.2\}$ be a nfbts with neutrosophic ideal L on X , $<\mu.\sigma.u>\epsilon\tau_i$ then we have :

(i) If < μ,σ,u > is PLNF*-closed then < μ,σ,u >* ≤ NFint(< μ,σ,u > ) ≤ NFcl(< μ,σ,u > ).

(ii) If < μ,σ,u > is PLNF*-dense- itself then NFLint(< μ,σ,u >) ≤ < μ,σ,u >*.

(iii) If < μ,σ,u > is PLNF*- perfect then NFLint(< μ,σ,u >) = NFcl(< μ,σ,u >) = < μ,σ,u >*.

**PROOF:** Obvious.

**THEOREM 4.2:** Given $(X,\tau_{i.}L).i\epsilon\{1.2\}$ be a nfbts with neutrosophic ideal $L = L_n$on X , $\mu.\sigma.u\epsilon I^X$ then we have the following:

< μ,σ,u > is neutrosophic fuzzy pairwse α - closed iff < μ,σ,u > is PLNF*- closed.

**PROOF:** It's clear.

**DEFİNİTİON 4.2: [7]** "Given $(X,\tau_i).i\epsilon\{1.2\}$ a nfbts with neutrosophic ideal L and $\xi.\rho.\theta\epsilon I^X$. $<\xi.\rho.\theta>$ is called neutrosophic fuzzy pairwise NFL-closed set if its complement is neutrosophic fuzzy NFL-open set. We will denote the family of neutrosophic fuzzy NFL-closed sets by NFPLC(X)".

**THEOREM 4.4:** Given $(X,\tau_i).i\epsilon\{1.2\}$ a nfbts with "neutrosophic ideal " L and $\xi.\rho.\theta\epsilon I^X$. $<\xi.\rho.\theta>$ η is neutrosophic fuzzy NFL − closed.

then $P((\text{Nfint} <\xi.\rho.\theta>)^{*}) \leq <\xi.\rho.\theta>$.

**PROOF.** It's clear.

**THEOREM 4.5:** Given (X,τ) be a nfbts with "neutrosophic ideal" L on X and $\xi.\rho.\theta\epsilon I^X$ such that $PL((\text{Nfint} <\xi.\rho.\theta>)^{*c}) = \text{Nfint}\ PL\left(\left(<\xi.\rho.\theta>^{c^{*}}\right)\right)$. then $<\xi.\rho.\theta>$ in NFPLC(X) iff $P((\text{Nfint}\ L <\xi.\rho.\theta>)^{*}) \leq <\xi.\rho.\theta>$

**PROOF.** (Necessity) Follows immedially from the above theorem. (Sufficiency). Let $PL((\text{Nfint} <\xi.\rho.\theta>)^{*}) \leq <\xi.\rho.\theta>$. then $<\xi.\rho.\theta>^{c} \leq (P(\text{Nfint} <\xi.\rho.\theta>))^{*c} = \text{NfintL} <\xi.\rho.\theta>^{c^{*}}$, from the hypothesis. Hence $<\xi.\rho.\theta>^{c}$ in NFPLO(X), Thus $<\xi.\rho.\theta>$ in NFPLC(X),

**COROLLARY 4.2:** For a nfbts $(X,\tau_i.L).i\epsilon\{1.2\}$ with "neutrosophic ideal" L on X the following holds:

(i) The union of "neutrosophic" NFPL-closed set and neutrosophic NFP-closed set may be neutrosophic NPL-closed set.

(ii) The union of "neutrosophic" NFPL-closed and neutrosophic NFPL-closed may be neutrosophic NFPL-closed.

**EXAMPLES 4.1:**

(i) The union of a "neutrosophic" NFPL-closed set and a neutrosophic NFP-closed set may be a neutrosophic NPL-closed set.

Let H1=A=<x,0.6,0.2,0.3> (NFPL-closed) Let F=Ac=<x,0.4,0.5,0.3> (NFP-closed)

H1∪F=A∪ Ac=<x, max (0.6,0.4), min (0.2,0.5), min (0.3,0.3)>=<x,0.6,0.2,0.3>=A.

Here, the union A is an NFPL-closed set. So, in this case, the union of an NFPL-closed set and an NFP-closed set is NFPL-closed.

However, to show it "may be", we should also consider a case where it is NOT NFPL-closed.

Let H1= Bc =<x,0.9,0.7,0.9> (NFPL-closed) Let F=A=<x,0.6,0.2,0.3> (NFP-closed)

H1∪F=<x, max (0.9,0.6), min (0.7,0.2), min (0.9,0.3)>=<x,0.9,0.2,0.3>.

Is <x,0.9,0.2,0.3> NFPL-closed? It is not in the list {∅N ,A,Ac,Bc,1N}.

Therefore, the union of an NFPL-closed set and an NFP-closed set may or may not be NFPL-closed. This example supports the statement.

(ii) The union of "neutrosophic" NFPL-closed and neutrosophic NFPL-closed may be neutrosophic NFPL-closed.

Let H1=A=<x,0.6,0.2,0.3> (NFPL-closed) Let H2 =Ac=<x,0.4,0.5,0.3> (NFPL-closed)

H1∪H2=A ∪ Ac=<x,0.6,0.2,0.3>=A.

Here, the union A is an NFPL-closed set. So, the union of two NFPL-closed sets is NFPL-closed in this case.

However, to show it "may be", we should also consider a case where it is NOT NFPL-closed.

Let H1=A=<x,0.6,0.2,0.3> (NFPL-closed) Let H2 =Bc=<x,0.9,0.7,0.9> (NFPL-closed)

H1∪H2=A∪ Bc=<x, max (0.6,0.9), min (0.2,0.7), min (0.3,0.9)>=<x,0.9,0.2,0.3>.

As we saw before, <x,0.9,0.2,0.3> is not in the list of NFPL-closed sets {∅N,A,Ac,Bc,1N}.

Therefore, the union of two NFPL-closed sets may or may not be NFPL-closed. This example supports the statement.

## 5. A PKI BASED SECURITY MODEL FOR MANET USING NEUTROSOPHIC FUZZY LOCAL FUNCTION

The applications of previous concepts are seems that they are related to some mathematical topics such as "neutrosophic fuzzy bitopological spaces", neutrosophic fuzzy local functions and neutrosophic fuzzy ideals. These topics may have some potential uses in fields such as logic, decision-making, artificial intelligence and geographic information systems.

**EXAMPLE 5.1** "A PKI based Security Model for MANET" using "Neutrosophic fuzzy local function" and proposes a neutrosophic fuzzy topology security scheme based on Public Key infrastructure for distributing session keys between nodes. The results shows that the using of neutrosophic fuzzy based security can enhance the security of (MANETs).

The following Table 5.1 Let A,B and C are neutrosophic fuzzy Sets for Security Data and we define the neutrosophic Topological Ideal Security Space.

*Table 5.1.*

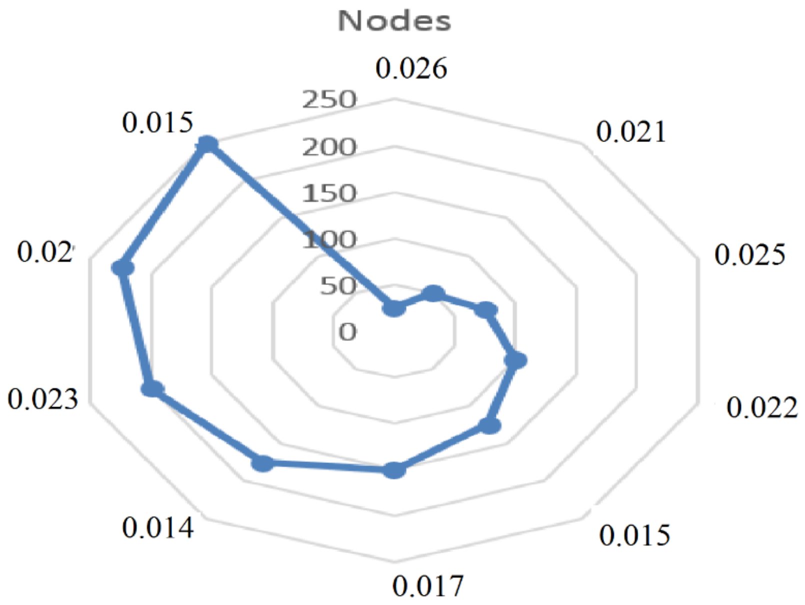
| No.Nodes | A=ASL(NS) | B=KCR(NS) | C=PKI(NS) |
|---|---|---|---|
| 25 | <0.026, 0.034, 0.94> | <0.95, 0.93, 0.07> | <0.15, 0.85, 0.15> |
| 50 | <0.021, 0.036, 0.943> | <0.021, 0.036, 0.943> | <0.2, 0.85, 0.15> |
| 75 | <0.025, 0.038, 0.937> | <0.95, 0.85, 0.15> | <0.23, 0.85, 0.15> |
| 100 | <0.022, 0.038, 0.939> | <0.96, 0.92, 0.08> | <0.26, 0.92, 0.08> |
| 125 | <0.015, 0.004, 0.981> | <0.96, 0.93, 0.07> | <0.3, 0.93, 0.07> |
| 150 | <0.017, 0.004, 0.979> | <0.96, 0.94, 0.06> | <0.32, 0.94, 0.06> |
| 175 | <0.014, 0.004, 0.982> | <0.96, 0.94, 0.06> | <0.36, 0.94, 0.06> |
| 200 | <0.023, 0.004, 0.973> | <0.96, 0.94, 0.06> | <0.4, 0.94, 0.06> |
| 225 | <0.02, 0.004, 0.976> | <0.02, 0.004, 0.976> | <0.44, 0.94, 0.06> |
| 250 | <0.015, 0.004, 0.981> | <0.96, 0.94, 0.06> | <0.45, 0.94, 0.06> |

.

Where $NFT = \{0_N, X_N, A, B, C\}$, $NFL = \{X_N, 0_N, A\}$, we calculate the following neutrosophic local function for the security "neutrosophic fuzzy" data A,B,C and D, where D represent the new security neutrosophic fuzzy data in the object neutrosophic topological Ideal Security Space (X,NFt,NL) . The following Table 5.2 represents the neutrosophic local function $A^*, B^*, D^*$ for NFTL(X) .

*Table 5.2*

| No.Nodes | $A^*$=ASL(NS) | $B^*$=KCR(NS) | $D^*$ |
|---|---|---|---|
| *25* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.026, 0.93, 0.94>* |
| *50* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.021, 0.036, 0.943>* |
| *75* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.025, 0.85, 0.937>* |
| *100* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.022, 0.92, 0.939>* |
| *125* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.015, 0.93, 0.981>* |
| *150* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.017, 0.94, 0.979>* |
| *175* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.014, 0.94, 0.982>* |
| *200* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.023, , 0.94, 0.973>* |
| *225* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.02, 0.004, 0.976>* |
| *250* | *<0, 0, 0>* | *<0, 0, 0>* | *<0.015, 0.94, 0.981>* |




*Not that : The class $D^*$ is largest security degree for the system such that $A^*$ contained D and $D^*$ neutrosophic subset for NFCL(D). This example illustrates the strength of neutrosophic topological Ideal Security Space complexity in safety.*

*Table 5.3, ASL of membership vs., non- membership and indeterminacy classification*

| No. Nodes | 25 | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Percentage Average of Classification** | 0.026 | 0.021 | 0.025 | 0.022 | 0.015 | 0.017 | 0.014 | 0.023 | 0.02 | 0.015 |
| **Percentage Average of non Classification** | 0.034 | 0.036 | 0.038 | 0.038 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| **İndeterminacy** | 0.94 | 0.943 | 0.937 | 0.939 | 0.981 | 0.979 | 0.982 | 0.973 | 0.976 | 0.981 |

*Table 5.1. The first table examines the relationship between the assertion scores by the neutrosophic functions of the three Neutrosophic fuzzy classes A, B, and C and found the degrees of assertiveness of type B higher than the degrees of certainty of both types A and C, and a combination of the certainty degrees between type B and type C.*

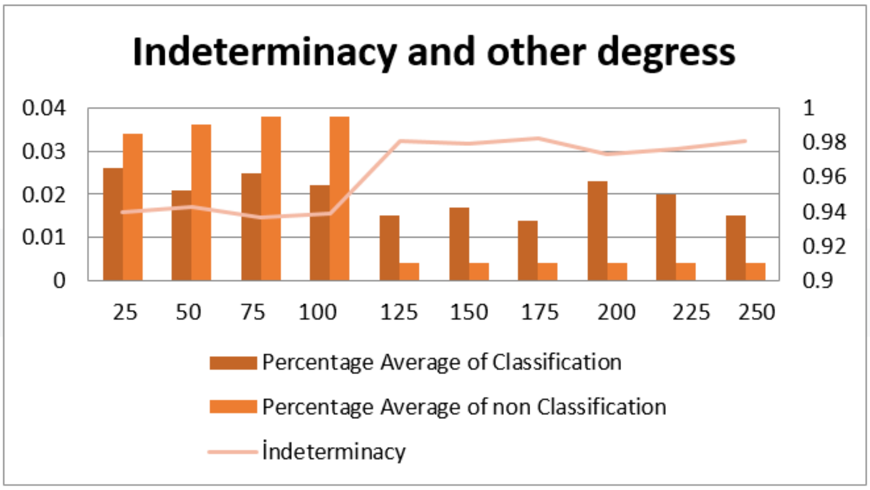
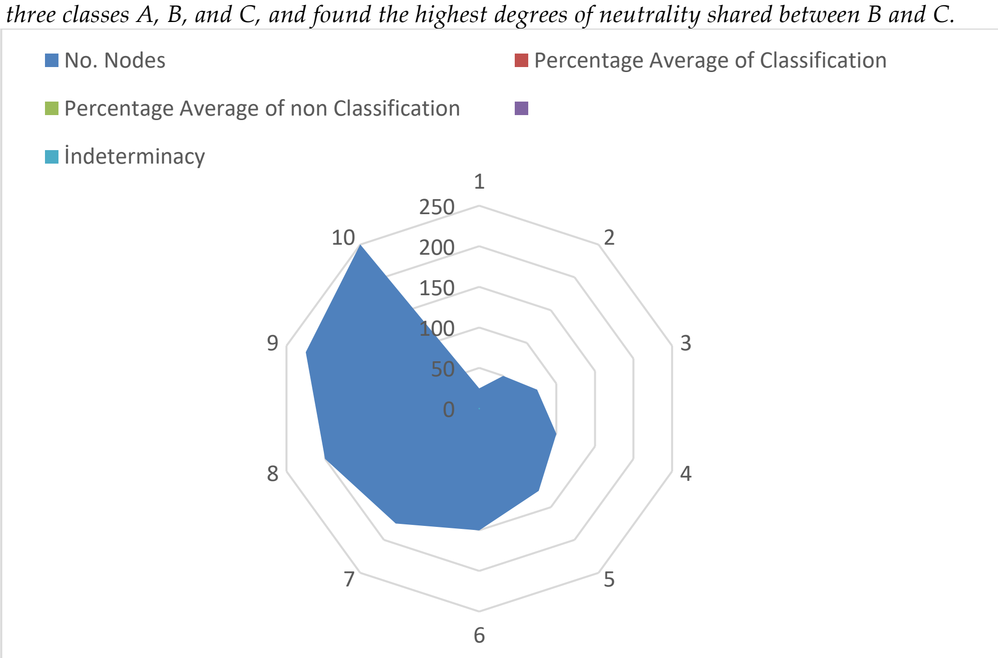



*Table 5.4, KCR of membership, non-membership and indeterminacy (neutrosophic Fuzzy classifiers)*

| No. Nodes | 25 | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Non-membership Classification** | 0.95 | 0.93 | 0.95 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 |
| **membership Classification** | 0.93 | 0.9 | 0.85 | 0.92 | 0.93 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 |
| **İndeterminacy** | 0.07 | 0.1 | 0.15 | 0.08 | 0.07 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |

*Table 5.2 The second table completes the relationship between the degrees of neutrality by means of the neutrosophic fuzzy functions of the three classes A, B, and C, and found the highest degrees of neutrality shared between B and C.*



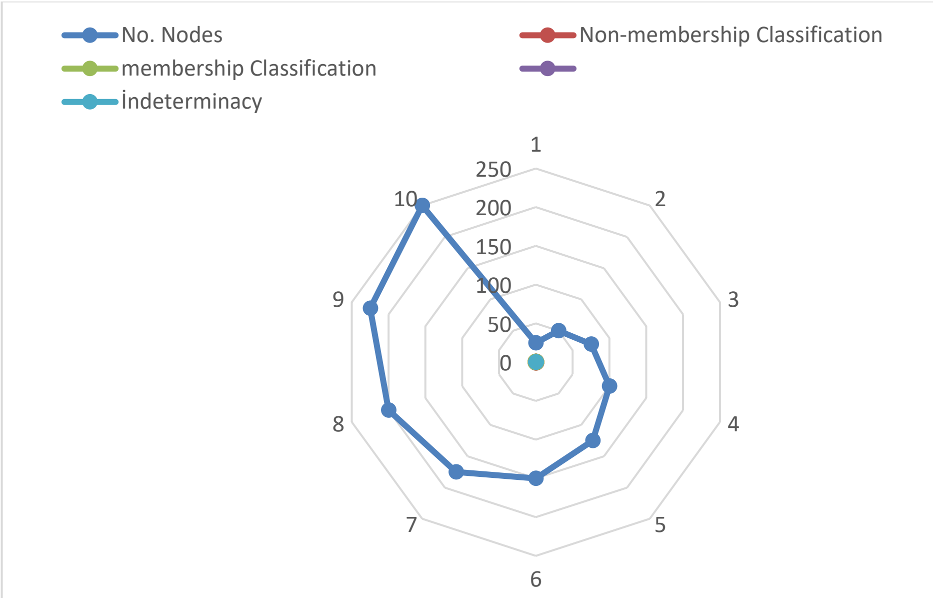
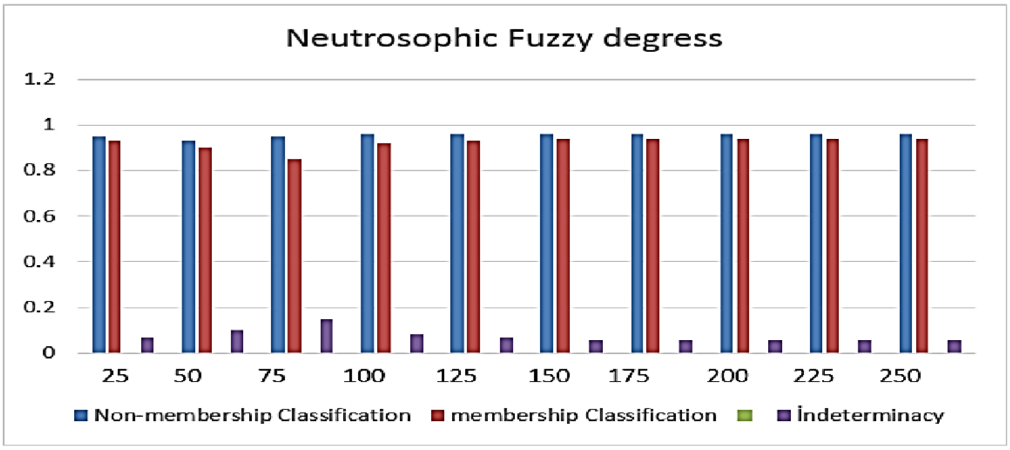



*Table 5.5, security of PKI vs, non-PKI and indeterminacy*

| No. Nodes | 25 | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 |
|---|---|---|---|---|---|---|---|---|---|---|
| Non-PKI | 0.15 | 0.2 | 0.23 | 0.26 | 0.3 | 0.32 | 0.36 | 0.4 | 0.44 | 0.45 |
| PKI | 0.85 | 0.85 | 0.85 | 0.92 | 0.93 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 |
| indeterminacy | 0.15 | 0.15 | 0.15 | 0.08 | 0.07 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |



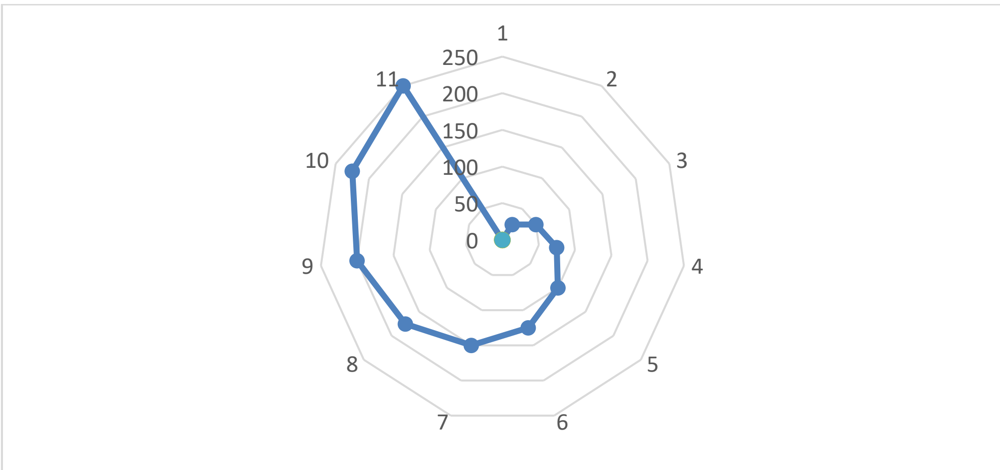




*Table 5.4 The third table shows the relationship between the degrees of uncertainty by the neutrosophic fuzzy functions of the three classes A, B, and C and found the lowest degrees of uncertainty are common between B, C and Class B in common with Class A in only two nodes.*

*Table 5.6.*

| No.Nodes | A=ASL(NS) | B=KCR(NS) | C=PKI(NS) |
|---|---|---|---|
| 25 | <0.026, 0.034, 0.94> | <0.95, 0.93, 0.07> | <0.15, 0.85, 0.15> |
| 50 | <0.021, 0.036, 0.943> | <0.021, 0.036, 0.943> | <0.2, 0.85, 0.15> |
| 75 | <0.025, 0.038, 0.937> | <0.95, 0.85, 0.15> | <0.23, 0.85, 0.15> |
| 100 | <0.022, 0.038, 0.939> | <0.96, 0.92, 0.08> | <0.26, 0.92, 0.08> |
| 125 | <0.015, 0.004, 0.981> | <0.96, 0.93, 0.07> | <0.3, 0.93, 0.07> |
| 150 | <0.017, 0.004, 0.979> | <0.96, 0.94, 0.06> | <0.32, 0.94, 0.06> |
| 175 | <0.014, 0.004, 0.982> | <0.96, 0.94, 0.06> | <0.36, 0.94, 0.06> |
| 200 | <0.023, 0.004, 0.973> | <0.96, 0.94, 0.06> | <0.4, 0.94, 0.06> |
| 225 | <0.02, 0.004, 0.976> | <0.02, 0.004, 0.976> | <0.44, 0.94, 0.06> |
| 250 | <0.015, 0.004, 0.981> | <0.96, 0.94, 0.06> | <0.45, 0.94, 0.06> |

*In the three Tables 5.4, 5.5 and 5.6. of local neutrosophic fuzzy functions, we find that D * represents the highest degree of restlessness in cases of certainty, neutrality and uncertainty.*

## 6. SOME NEUTROSOPHIC FUZZY CODES.

A neutrosophic local function is a function that maps "a neutrosophic Fuzzy topological space" to another "neutrosophic topological space", preserving some properties of the original space. A neutrosophic topological space is a generalization of a fuzzy topological space, where each element has a degree of truth, falsity and indeterminacy3.
The following code for a neutrosophic local function in Python,

```
# Define a class for neutrosophic sets
class NeutrosophicSet:
  def __init__(self, elements, truth, falsity, indeterminacy):
    # elements is a list of elements in the set
    # truth, falsity and indeterminacy are dictionaries that map each
element to its degree of truth, falsity and indeterminacy
    self.elements = elements
    self.truth = truth
    self.falsity = falsity
    self.indeterminacy = indeterminacy
  def __str__(self):
    # Return a string representation of the set
    result = "{"
    for element in self.elements:
      result += str(element) + ": (" + str(self.truth[element]) + ", " +
str(self.falsity[element]) + ", " + str(self.indeterminacy[element]) + "),
"
    result = result[:-2] + "}"
    return result
# Define a class for neutrosophic topological spaces
class NeutrosophicTopologicalSpace:
  def __init__(self, base_set, open_sets):
    # base_set is a NeutrosophicSet that represents the underlying set of
the space
    # open_sets is a list of NeutrosophicSets that represents the open
sets of the space
    self.base_set = base_set
    self.open_sets = open_sets
```

```
  def __str__(self):
    # Return a string representation of the space
    result = "Base set: " + str(self.base_set) + "\n"
    result += "Open sets: \n"
    for open_set in self.open_sets:
      result += str(open_set) + "\n"
    return result
# Define a function that checks if a function is a neutrosophic local
function
def is_neutrosophic_local_function(f, X, Y):
  # f is a function that maps X to Y, where X and Y are
NeutrosophicTopologicalSpaces
  # Return True if f is a neutrosophic local function, False otherwise
  # A neutrosophic local function satisfies the following conditions:
  # 1. For any x in X.base_set, f(x) belongs to Y.base_set
  # 2. For any x in X.base_set and any U in Y.open_sets, if f(x) belongs
to U, then there exists V in X.open_sets such that x belongs to V and f(V)
is a subset of U
  # 3. For any x in X.base_set and any U in Y.open_sets, if f(x) does not
belong to U, then there exists V in X.open_sets such that x belongs to V
and f(V) does not intersect U
  for x in X.base_set.elements:
    # Check condition 1
    if f(x) not in Y.base_set.elements:
      return False
    for U in Y.open_sets:
      # Check condition 2
      if f(x) in U.elements:
        found_V = False
        for V in X.open_sets:
          if x in V.elements and all(f(v) in U.elements for v in
V.elements):
            found_V = True
            break
        if not found_V:
          return False
      # Check condition 3
      else:
        found_V = False
        for V in X.open_sets:
          if x in V.elements and all(f(v) not in U.elements for v in
V.elements):
            found_V = True
            break
        if not found_V:
          return False
  return True
```

```
# Example: Define two neutrosophic topological spaces and a function between them
# Define the base sets
X_base = Neutrosophic Set([1, 2, 3], {1: 0.8, 2: 0.6, 3: 0.4}, {1: 0.1,
Received message. A neutrosophic local function is
```

The code consists of three parts: a class for neutrosophic sets, a class for neutrosophic topological spaces, and a function that checks if a function is a neutrosophic local function.

The class for neutrosophic sets defines a data structure that can store the elements of a neutrosophic set and their degrees of truth, falsity and indeterminacy. The constructor takes four arguments: a list of elements, and three dictionaries that map each element to its degree of truth, falsity and indeterminacy. The class also has a method that returns a string representation of the set.

The class for neutrosophic topological spaces defines a data structure that can store the base set and the open sets of a neutrosophic topological space. The constructor takes two arguments: a neutrosophic set that represents the base set, and a list of neutrosophic sets that represents the open sets. The class also has a method that returns a string representation of the space.

The function that checks if a function is a neutrosophic local function takes three arguments: a function that maps one neutrosophic topological space to another, and the two spaces. The function returns True if the function satisfies the definition of a neutrosophic local function, and False otherwise. The definition of a neutrosophic local function is based on the paper by Salama and Smarandache, and it has three conditions:

1. For any x in the base set of the first space, f(x) belongs to the base set of the second space.
2. For any x in the base set of the first space and any open set U in the second space, if f(x) belongs to U, then there exists an open set V in the first space such that x belongs to V and f(V) is a subset of U.
3. For any x in the base set of the first space and any open set U in the second space, if f(x) does not belong to U, then there exists an open set V in the first space such that x belongs to V and f(V) does not intersect U.

The function iterates over all elements in the base set of the first space, and checks each condition using nested loops and logical expressions. If any condition is violated, it returns False. Otherwise, it returns True.

**The following example shows of how to use the code.**

Let's say we have two "neutrosophic topological spaces" X and Y, and a function f that maps X to Y. We want to check if f is a neutrosophic local function.

First, we need to define the base sets and the open sets of X and Y using the NeutrosophicSet class. For example:

```
# Define the base sets
X_base = NeutrosophicSet([1, 2, 3], {1: 0.8, 2: 0.6, 3: 0.4}, {1: 0.1, 2: 0.2, 3: 0.3}, {1: 0.1, 2: 0.2, 3: 0.3})
Y_base = NeutrosophicSet([4, 5, 6], {4: 0.7, 5: 0.5, 6: 0.3}, {4: 0.2, 5: 0.3, 6: 0.4}, {4: 0.1, 5: 0.2, 6: 0.3})

# Define the open sets
X_open = [NeutrosophicSet([1], {1: 1}, {1: 0}, {1: 0}), NeutrosophicSet([2], {2: 1}, {2: 0}, {2: 0}), NeutrosophicSet([3], {3: 1}, {3: 0}, {3: 0}), NeutrosophicSet([1,2], {1: 0.9, 2: 0.9}, {1: 0.05, 2: 0.05}, {1: 0.05, 2: 0.05})]

Y_open = [NeutrosophicSet([4], {4: 1}, {4: 0}, {4: 0}), NeutrosophicSet([5], {5: 1}, {5: 0}, {5: 0}), NeutrosophicSet([6], {6: 1}, {6: 0}, {6: 0}), NeutrosophicSet([4,5], {4:
Received message. Sure, I can give you an example of how to use the code.
```

Let's say we have two "neutrosophic topological space".

Next, we need to define the "neutrosophic topological spaces" X and Y using the "Neutrosophic Topological Space" class. For example:

```
#Define the neutrosophic topological spaces
X = NeutrosophicTopologicalSpace(X_base, X_open)
Y = NeutrosophicTopologicalSpace(Y_base, Y_open)
```

Then, we need to define the function f that maps X to Y. For example:

```
# Define the function f
def f(x):
  # A simple function that maps 1 to 4, 2 to 5 and 3 to 6
  return x + 3
```

Finally, we can use the is "neutrosophic local function" to check if f is a "neutrosophic local function". For example:

```
# Check if f is a neutrosophic local function
print(is_neutrosophic_local_function(f, X, Y))
```

This should print True, because f satisfies the definition of a "neutrosophic local function". I hope this example helps you use the code.

Received message. Okay, I will continue the example. Next, we need to define the neutrosophic top.

Next, we need to define the "neutrosophic topological spaces" X and Y using the "Neutrosophic Topological Space class". For example:

```
# Define the neutrosophic topological spaces
X = NeutrosophicTopologicalSpace(X_base, X_open)
Y = NeutrosophicTopologicalSpace(Y_base, Y_open)
```

Then, we need to define the function f that maps X to Y. For example:

```
# Define the function f
def f(x):
  # A simple function that maps 1 to 4, 2 to 5 and 3 to 6
  return x + 3
```

Finally, we can use the is "neutrosophic local function" to check if f is a neutrosophic local function. For example:

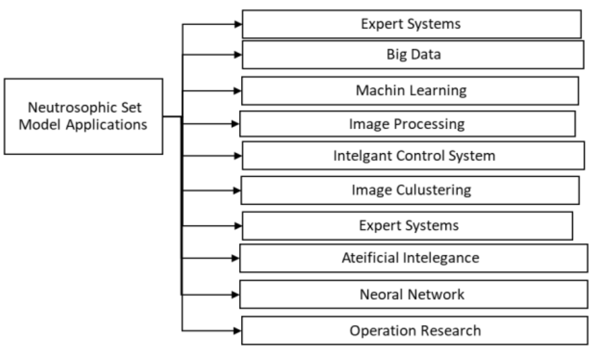
```
# Check if f is a neutrosophic local function
print(is_neutrosophic_local_function(f, X, Y))
```

This should print True, because f satisfies the definition of a neutrosophic local function.

I hope this example helps you use the code.

## 7. Conclusions and Future Work

For neutrosophic set model, there are various applications [3,4] in different areas as shown in the Following figure.




The concepts of the sets and functions in "neutrosophic bitopological spaces" are highly developed and several characterizations have already been obtained.

In this work, we have proposed a novel security model based on neutrosophic fuzzy bitopological ideals. The main motivation of our study is to address the challenges and issues of security in some applications that require two levels of security or encryption, such as cloud computing, wireless sensor networks, cryptography, etc.

In this work, some new applications on neutrosophic bitopological ideal spaces (NBIS) were introduced and studied. Relationships between the new neutrosophic fuzzy pairwise notions and their other relevant classes were investigated. Lastly, we introduced some applications by using neutrosophic bitopological ideal spaces on PKI based Security Model for MANET by Neutrosophic fuzzy local function and proposes a neutrosophic fuzzy bitopology security system based on Public Key infrastructure for distributing session keys between nodes.

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