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SOME ISSUES ON TOPOLOGICAL OPERATORS VIA IDEALS

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ABSTRACT. In a topological structure (X_1,τ) any ideal I of a subset of X induces a new topology τ^1 (I). If Y is a regular structure, then under some assumptions on I, for any upper τ^*- quasi continuous multivalued map $F : X_1 \to P(Y)\setminus\emptyset$,the sets of all points at which F is lower quasi- continuous (lower semi-continuous with respect to τ^* *or* τ coincides). If F is compact- valued lower τ^* - quasi continuous, then the symmetrical result holds (J.Ewert [8]). The paper concerns operators in ideal topological structures. Some properties and characterizations of the set operator $(\ )^{*s}$ are investigated and explored. We define, explore, and analyze nano semi- local function as well as some of its characteristics. Finally, we design a graph of vital places in our city with regard to ideals.

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KEYWORDS: Local functions; Ideal , nano semi Local functions.

1. INTRODUCTION

Kuratowski has studied ideals in topological structures since 1930 [1]. The study written by Vaidyanathaswamy [2] in 1945 helped to establish the topic's significance.

In the rich and nuanced landscape of topology, the concept of ideals plays a pivotal role in shaping and refining various topological structures. The interplay between ideals and topology gives rise to a multitude of new perspectives and tools, enhancing our ability to analyze and manipulate topological spaces. This paper, "Topological Operators Via Ideals," embarks on an in-depth exploration of how ideals influence and define topological operators, revealing the profound impacts and applications of these interactions [21-28].

An ideal I of a subset of a topological space (X_1,τ) introduces a modified topology $\tau(I)$, fundamentally altering the way we perceive continuity, convergence, and other topological properties. By leveraging these induced topologies, we can develop a richer understanding of multivalued maps, set operators, and function characteristics within these redefined spaces. The investigation of upper τ^* - quasi continuous multivalued maps $F : X_1 \to P(Y)\setminus\emptyset$ is a regular structure, stands at the forefront of this exploration. We aim to discern the conditions under which these maps exhibit lower quasi-continuity, specifically lower semi-continuity concerning τ^* *or* τ, thereby enhancing our comprehension of continuous behavior in ideal-induced topologies.

Building upon the foundational work of J. Ewert, this paper further explores the symmetrical properties of compact-valued lower τ^* -quasi continuous maps, offering a comprehensive view of continuity in the context of ideals. This dual examination of upper and lower quasi-continuity sheds light on the intricate balance maintained by these topological constructs.

In addition to the study of continuity, this paper delves into the properties and characterizations of the set operator $F : X_1 \to P(Y)\setminus\emptyset$ within ideal topological structures. By thoroughly analyzing this operator, we uncover new dimensions of its behavior and implications, contributing to a deeper theoretical understanding of topological operations influenced by ideals.

The introduction of the nano semi-local function marks another significant advancement in this research. By defining and examining these functions, we reveal their unique characteristics and potential applications, further expanding the toolkit available for topological analysis.

To illustrate the practical relevance of these theoretical findings, we design a graph representing vital places in a city, informed by the underlying ideals. This practical application not only demonstrates the real-world utility of ideal topological structures but also bridges the gap between abstract theoretical concepts and tangible implementations.

In summary, "Topological Operators Via Ideals" offers a thorough and insightful investigation into the ways ideals transform topological operators. Through a blend of theoretical exploration and practical application, this paper enriches the field of topology with new properties, characterizations, and innovative approaches to understanding and utilizing topological spaces influenced by ideals.

Subsequently, numerous mathematicians employed the notation of ideals in topological structures. (see [3,4,10, 13,14,15,16,17]).

The use of ideals in general topology received a major boost by Newcomb in 1967 and by Berri, Porter and Stephenson in a surrey paper in 1968. The role of ideals in topological structures is a tool like filters. An ideal or dual filter on X_1 is a collection of non-empty subsets of X_1 that fulfills the requirements of heredity and finite additivity. A family $I \subset P(X_1)$ is called an ideal iff (i) $A \in I$ gives $P(A) \subseteq I$, and (ii) $A,B \in I$ gives $A \cup B \in I$. For examples, $\{\emptyset\}$, $P(X_1)$.

The families of all closed-discrete subsets I_{cd}, all meagre subsets I_m, all scattered subjects I_s, and all nowhere dense subsets I_n.The closed set of functions that take the form is defined as the local function of an ideal I with regard to when it is defined on P (X) as:

$K_1^*(\tau, I) = X_1 \backslash \cup \{G \in \tau : G \cap K_1 \in I\}(K_1 \subseteq X$) [2] or equivalently,

$K_1^*(\tau, I) = \{x \in X_1 : \forall G_x \in \tau_x , G_x \cap K_1 \notin I\}$, where $\tau_x = \{G \in \tau : x \in G\}$.

This closed set is written simply by $K_1^*(I)$ or even by K_1^* if there is no possibility of confusion. $K_1^* \subseteq Cl(K_1)$ is evident. It is quite easy to see that $K_1^*(\tau_2, I_2) \subseteq K_1^*(\tau_1, I_1)$

whenever $\tau_1 \subseteq \tau_2$ and $I_1 \subseteq I_2$.

The following fundamental characteristics of local function operator ( )*are also clearly demonstrated:

(1) $(K_1 \cup B)^* = K_1^* \cup B^*,\ E^* = \varnothing,\ (G \backslash E) \cap K_1^* \subseteq ((G \backslash E) \cap K_1)^*$,

(2) $(K_1^*)^* \subseteq K_1^*$ for any $K_1,B \subseteq X$, $G \in \tau$ *and* $E \in I$.

The family $\tau^*(I) = \{K_1 \subseteq X : K_1 \cap (X \backslash K_1)^* = \emptyset\}$ is a topology on X_1 better than τ. A subset K_1 is closed in ($X_1, \tau^*(I))$ iff $K_1^* \backslash K_1) = (X_1 \backslash K_1) \cap K_1^* = \varnothing$ iff $K_1^* \subseteq K_1$. Consequently, the closure and interior operators in $(X_1,\tau^*(I))$ satisfy easily, Cl $^*(K_1) = K_1 \cup K_1^*$ and $\text{Int}^*(K_1) = K_1 \backslash (X_1 \backslash K_1)^*$.

The topological structure $\tau^*(I)$ may lack interest due to its discrete nature, as exemplified by the following example.

For example, if X_1 be the set of natural numbers, and τ be defined as follows: $\tau = \{\emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \ldots\}$.

The concept of a nτ was defined by Lellis Thivagar and Carmel Richard [3] which was defined using approximations and boundary region of a subset of an universe using an *ER* on it and also they defined nclosed sets,n interior and n closure. The concept of nIτ structures was introduced by Parimala et al. [4] and investigated its characteristics and properties. This study investigates and explores various properties and characterizations of the set operator ( )^{*s}. This work examines operators in n*I*τs. We introduce the relationships between some weak forms of nopen sets in nτ structures and some weak forms of nopen sets in nIτ structures. Also, we point out that the class of npI open sets is properly places between the classes of nI-open and npreopen sets. We give a decomposition of nI-continuity by proving that a mapping $f_1 : (X_1,\tau,I) \to (Y_1,\sigma)$ is nI-continuous iff it is npI-continuous and n$*I$-continuous.

Before entering in our work, we mention the next definitions.

2. PRELIMINARIES

Throughout this entire article, we will refer to U_1 and V_1 as an initial universal sets, U/R and V/R are equivalence classes (*EC*), *R* and *R* are equivalence relations (*ER*), $X_1 \subseteq U_1, Y_1 \subseteq V_1$.

DEFINITION 2.1.

A subset A_1 of a topological structure (X_1,τ) is called

(i) Semi-open [7], if $A_1 \subseteq Cl(Int(A_1))$,

(ii) Semi-closed [5,6], if $X_1 \backslash A_1$ is semi- open.

The union of all semi-open sets contained in A is denoted as sInt (A) and is known as the semi-interior of A. The intersection of all semi-closed sets containing A is denoted by sCl(A) and is known as the semi-closure of A [5].

DEFINITION 2.1. [6]

Consider Fig. 1

Assume that (U_1,R) be an approximation structure and $X_1 \subseteq U_1$. Then:

The lower approximation ($L_{app}(X_1)$) and upper approximations ($H_R(X_1)$), since

Lapp(X1) = Sx∈*U*{x1 ∈ *U* : [x1]*R* ⊆ X1}, *HR*(X1) =

$Sx \in U\{x1 \in U : [x1]R \cap X1 \neq \phi\}$.
and the boundary region $B_R(X_1)$ that is
$BR(X1) = HR(X1) - Lapp(X1)$.

According to Pawlak's definition, X_1 is called a rough set if $H_R(X_1) \neq L_{app}(X_1)$.

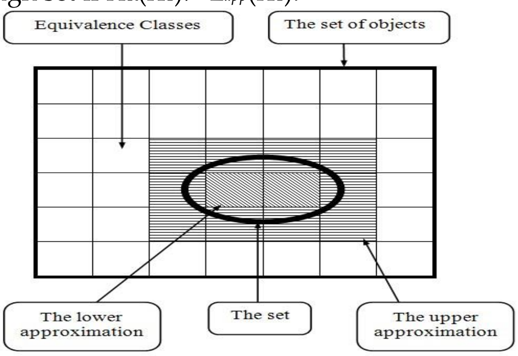

Fig. 1. A rough set in a rough approximation structure.

PROPOSITION 2.2. *([6])*

If (U_1,R) is an approximation structure and $X_1,Y_1 \subseteq U_1$, then we , have the next properties of Pawlak s rough sets:

(i) $L_{app}(X_1) \subseteq X_1 \subseteq H_R(X_1)$.

(ii) $L_{app}(\phi) = H_R(\phi) = \phi$ & $L_{app}(U_1) = H_R(U_1) = U_1$.

(iii) $H_R(X_1 \cup Y_1) = H_R(X_1) \cup H_R(Y_1)$. *(iv)* $H_R(X_1 \cap Y_1) \subseteq H_R(X_1) \cap H_R(Y_1)$.

(v) $Lapp(X1 \cup Y1) \supseteq Lapp(X1) \cup Lapp(Y1)$.

(vi) $L_{app}(X_1 \cap Y_1) = L_{app}(X_1) \cap L_{app}(Y_1)$ *(Multiplication)*. *(vii)* $L_{app}(X_1) \subseteq L_{app}(Y_1)$ *and* $H_R(X_1) \subseteq H_R(Y_1)$ *whenever* $X_1 \subseteq Y_1$

(viii) $H_R(X_1^c) = [H_R(X_1)]^c$ *and* $L_R(X_1^c) = [H_R(X_1)]^c$

(ix) $H_R(H_R(X_1)) = L_{app}(H_R(X_1)) = H_R(X_1)$.

(x) $Lapp(Lapp(X1)) = HR(Lapp(X1)) = Lapp(X1)$.

DEFINITION 2.3. [3]

A $n\tau$ on U_1 and $X_1 \subseteq U_1$, . Then $\tau_R(X_1) = \{U,\phi,L_R(X_1),H_R(X_1),B_R(X_1)\}$ that satisfies the next axioms:

(i) U_1 and $\phi \in \tau_R(X_1)$;

(ii) The union of the components of any subgroups of $\tau_R(X_1)$ is in $\tau_R(X_1)$;

(iii) The intersection of the components of any finite subgroups of $\tau_R(X_1)$ is in $\tau_R(X_1)$.

That is, $\tau_R(X_1)$ is a topology on U_1 called the ntopology on U_1 with respect to X_1 and the pair $(U_1,\tau_R(X_1))$ is called a $n\tau$ structure. The elements of $\tau_R(X_1)$ are called nopen sets in U_1 and the complement of a nopen set is called a nclosed set. The elements of $[\tau_R(X_1)]^c$ being called dual $n\tau$ of $\tau_R(X_1)$.

REMARK 2.4.

If $\tau_R(X_1)$ is $n\tau$ on U_1 with respect to X, then Lellis Thivagar and Carmel Richard [3] observed that the family $\beta = \{U_1,L_R(X_1),B_R(X_1)\}$ is the basis for $\tau_R(X_1)$.

REMARK 2.5.

Let $(U_1,\tau_R(X_1))$ be a $n\tau$ structure with respect to X_1 such that $X_1 \subseteq U_1$ and R be an ER on U_1. Then U_1/R denotes the family of EC of U_1 by R.

DEFINITION 2.6. ([3], [7], [9])

If $(U,\tau_R(X_1))$ be a $n\tau$ structure and $S \subseteq U_1$. , then S is said to be:

(i) N*regular open if* $S = nInt(nCl(S))$,

(ii) $n\alpha - open$ *if* $S \subseteq nInt(nCl(nInt(S)))$,

(iii) *nsemi – open if* $S \subseteq nCl(nInt(S))$,

(iv) *npreopen if* $S \subseteq nInt(nCl(S))$,

(v) $n\gamma - open$ (or $nb - open$) *if* $S \subseteq nCl(nInt(S)) \cup nInt(nCl(S))$, (vi) $n\beta - open$ *if* $S \subseteq nCl(nInt(nCl(S)))$.

The complement of a nregular open (resp. $n\alpha$-open, nano semi-open, npreopen, $n\gamma - open$, $n\beta$-open) set is called a nregular closed (resp. $n\alpha$-closed, n semi-closed, npreclosed, $n\gamma - closed$, $n\beta$-closed) set. The family of all nsemi-open sets of a $n\tau$ structure $(U_1,\tau_R(X_1),I)$ is denoted by $nSO(U_1,X_1)$.

DEFINITION 2.7.

A subset S of a $nI\tau$ structure $(U_1,\tau_R(X_1),I)$ is called npI-open if $S \subseteq nInt(nCl^*(S))$.

We denote by $nPIO(U_1,\tau_R(X_1),I)$ the family of all npI open subsets of $(U_1,\tau_R(X_1),I)$ or simply write $nPIO(U,\tau_R(X_1))$ or $nPIO(U)$ when there is no possibility of being mistaken for the ideal. We call a subset $S \subseteq (U_1,\tau_R(X_1),I)$ npI-closed if its complement is npI open.

3. SOME CHARACTERIZATIONS OF SEMI- LOCAL FUNCTIONS

PROPOSITION 3.1.

The following two properties are equivalents for any ideal topological structure (X_1,τ,I) *(P_1)* $K_1 \in I$ *iff* $K_1^*(I) = \varnothing$ *iff* $K_1 \cap K_1{}^*(I) = \varnothing$.

(P_2) If $\{K_{1\propto} : \propto \in \nabla\}$ *is a family of sets belonging to I and each* $K_{1\propto}$ *is an open set in the subspace* $\cup\{K_{1\propto} : \propto \in \nabla\}$, *then* $\cup\{K_{1\propto} : \propto \in \nabla\} \in I$.

PROOF (NECESSITY)

Let(P_1) be satisfied and let $\{K_{1\propto} : \propto \in \nabla\}$ be a family of sets belonging to **I** since each $K_{1\propto}$ is open in $\cup\{K_{1\propto} : \propto \in \nabla\}$. For each point $x \in \cup\{K_{1\propto} : \propto \in \nabla\}$ there exist $\propto_x \in \nabla$ and on open set G in (X_1,τ) since $x \in G \cap (\cup\{K_{1\propto} : \propto \in \nabla\}) = K_{1\propto} \in I$ .Thus $(\cup\{K_{1\propto} : \propto \in \nabla\}) \cap (\cup\{K_{1\propto} : \propto \in \nabla\})^* = \emptyset$, which implies $\cup\{K_{1\propto} : \propto \in \nabla\} \in I$.

(SUFFICIENCY)

Let (P_2) hold. Since the implications $K_1 \in I \Rightarrow K_1^*(I) = \varnothing$ and

$K_1^*(I) = \varnothing \Rightarrow K_1 \cap K_1{}^*(I) = \varnothing$ are true, it suffices to prove $K_1 \cap K_1{}^*(I) = \emptyset \Rightarrow K_1 \in I$. If $K_1 \cap K_1{}^*(I) = \varnothing$ holds, $\forall x \in K_1$ has an open neighborhood G_x since $G_x \cap K_1 \in I$. So, $\{G_x \cap K_1 : x \in K_1\}$ is a family of sets belonging to I and each $G_x \cap K_1$ is open in the subspace $\cup\{G_x \cap K_1 : x \in K_1\}$. Form the Kssumption we obtain $K_1 = \cup\{G_x \cap K_1 : x \in K_1\} \in I$, Completed proof.

REMARK 2.1.

If an ideal I satisfies (P_1) , then $\beta(I) = \tau^*(I)$ (where $\beta(I) = \{G\backslash E : G \in \tau, E \in I\}$ is a base of the topology $\tau^*(I)$ in X_1). [8], this equality is not necessary, as shown by the following example due to Ewert [8].

EXAMPLE2.1.

If (R,U) is usual topology and I is the ideal of all bounded subsets of Q⊆R, then $R\backslash Q = \cup\{(-n,n) \backslash [-n,n] \cap Q : n = 1,2,\ldots,\infty\} \in \tau^*(I)$ and $R\backslash Q \in / \beta(I)$.

DEFINITION 3.1

The semi-local function [14] of an ideal **I** with respect to τ is defined on P(X) as: -

$K_1{}^{*s}(I,\tau) = \{x\epsilon X : G\cap K_1 \notin I \text{ for each } G\epsilon SO(X_1,\tau)\}$. writing simply $K_1^{*s}(I)$ for $K_1{}^{*s}(I,\tau)$.

REMARK 3.1

If an ideal I has the property (P_1), then $K_1 \cup K_1{}^{*s}(I) = sCl^*(K_1)$ for each set $A \subseteq X_1$.

PROPOSITION 3.1

Let (X_1,τ,I) be an ideal topological structure and $K_1 \subseteq X_1$ thus.

(1) $K_1^{*s}(I) \subseteq K_1^*(I), \forall K_1 \subseteq X_1$.

(2) $K_1^{*s}(I) = K_1^*(I)$ if $\tau = SO(X_1,\tau)\}$.

(3) If $K_1 \in I$, then $K_1^*(I) = \varnothing$.

(4) In general, neither $\beta \subseteq K_1^{*s}(I)$ nor $K_1^{*s}(I) \subseteq K_1$.

PROOF

The statements (2) - (4) are are a direct result of the definitions. now, let $x \in K_1^{*s}(I)$. Then from Definition 3.1, $K_1 \cap G \,/\!\in I$ for each $G\epsilon SO(X_1,\tau)$. Since each open set is semi- open. Therefore $x \in K_1^*(I)$ and (1) is proved.

EXAMPLE3.1.

The converse of Statement (1) from Proposition 3.1 is not generally true. Let $X_1 = \{1,2,3,4\}$ and $\tau = \{X,\emptyset,\{1,2\}\}$ with $I = \{\emptyset,\{1\},\{2\},\{1,2\}\}$. Set $K_1 = \{1,2\}$, then $K_1^*(I) = \{3,4\} = Cl(K_1^*(I))$ and $K_1^{*s}(I) = \{4\} = sCl(K_1^{*s}(I))$.

LEMMA 3.1

If an ideal I in a topological structure (X_1,τ) satisfies property (P_1), then $\forall K_1 \subseteq X_1$, the sets $Cl^*(K_1)\backslash sCl^*(K_1)$ and $sInt^*(K_1)\backslash Int^*(K_1)$ are nowhere dense in (X_1,τ).

PROOF

Let $x \in Int(K_1{}^*(I))$ and let $x \in W$ is a semi- open. There for $G = Int(K_1{}^*(I)) \cap Int(W)$ is a non-empty open set, so $G \cap K_1 \in / I$, which implies $x \in K_1^{*s}(I)$. Thus we have $Int(K_1^*(I) \subset K_1^{*s}(I)$. Since $K_1^*(I)$ is a closed set [8] and $K_1^*(I)\backslash K_1^{*s}(I) \subset K_1^*(I)\backslash Int(K_1{}^*(I))$. Thus, it follows the difference $K_1^*(I)\backslash K_1^{*s}(I)$ is nowhere dense. now, we have $Cl^*(K_1)\backslash sCl^*(K_1) = (K_1 \cup K_1^*(I))\backslash(K_1 \cup K_1^{*s}(I)) \subset K_1^*(I)\backslash K_1^{*s}(I)$; so, $Cl^*(K_1)\backslash sCl^*(K_1)$ is nowhere dense.

For each subset K_1 of (X_1,τ) ,the formula below is true: $sInt(K_1) = X_1\backslash sCl(X\backslash K_1)$; [5].

Thus, we obtain, $sInt^*(K_1)\backslash Int^*(K_1) = (X_1\backslash sCl^*(X_1\backslash K_1)\backslash(X_1\backslash Cl^*(X_1\backslash K_1)) = Cl^*(X_1\backslash A)\backslash sCl^*(X_1\backslash K_1)$, so, $sInt^*(K_1)\backslash Int^*(K_1)$ is a nowhere dense set in (X_1, τ).

PROPOSITION 3.2

Let (X_1, τ, I) be an ideal topological structure and $K_1, B \subseteq X_1$.

The following characteristics hold for the semi-local function:

(a) $K_1^{*s}(I) = sCl(K_1^{*s}(I)) \subseteq sCl(K_1)$;

(b) $K_1^{*s}(I) \subset K_1^*(I)$;

(c) If $K_1 \subseteq B$, then $K_1^{*s}(I) \subseteq B^{*s}(I)$;

(d) $(K_1 \cup B)^{*s}(I) = K_1^{*s}(I) \cup B^{*s}(I)$;

(e) $(K_1^{*s})^{*s}(I) \subseteq K_1^{*s}(I)$;

(f) $K_1^{*s}(I)$ is a semi-closed set in X_1.

(g) If $E \in I$, then $(K_1\backslash E)^{*s}(I) \subset K_1^{*s}(I) = (K_1 \cup E)^{*s}(I)$;

(h) $(K_1)^{*s}(I)\backslash B^{*s}(I) = (K_1\backslash B)^{*s}(I)\backslash B^{*s}(I) \subset (K_1\backslash B)^{*s}(I)$;

(i) If $G \in \tau$, then $G \cap K_1^*(I) = G \cap (G \cap K_1)^{*s}(I) \subseteq (G \cap K_1)^{*s}(I)$;

(j) If $I = \{\emptyset\}$, then $K_1^{*s}(I) = sCl(K_1)$;

(k) If $I = P(X_1)$, then $K_1^*(I) = \varnothing$, gives $K_1^{*s}(I) = \varnothing$, *for each* $K_1 \subseteq X_1$;

PROOF

(a) In general, $(K_1)^{*s}(I) \subset sCl(K_1^{*s}(I))$.

Let $x\epsilon\ sCl(K_1^{*s}(I))$. Then $(K_1)^{*s}(I) \cap G \neq \emptyset$ for each $G \in SO(X,I)$.Therefore, there exists a point $y \in (K_1)^{*s}(I) \cap G$ and $G \in SO(X,y)$. Since $y \in K_1{}^{*s}(I), K_1 \cap G \notin I$ and hence $x \in K_1^{*s}(I)$.

Thus, we have $sCl(K_1^{*s}(I)) \subset K_1^{*s}(I)$.

Also, let $x \in sCl\,(K_1^{*s}(I)) = K_1^{*s}(I)$, then $G \cap$

$K_1 \in/ I$ for each $G \epsilon SO(X_1,x)$.
This implies $G \cap K_1 \in/ \emptyset$, for each $G \epsilon SO(X_1,x)$. Hence, $x \epsilon\, sCl(K_1)$. This proves
$K_1^{*s}(I) = sCl(K_1^{*s}(I)) \subseteq sCl(K_1)$.

(b) Follows from the definition.

(c) Consider x /∈ $B^{*s}(I)$, there for there is $G \epsilon SO(X_1,x)$. Since $G \cap B \in I$. Since $K_1 \subset B$, $G \cap K_1 \in I$ and $x \notin K_1^{*s}(I)$. Which finishes the proof.

(d) From (c), we have $K_1^{*s}(I) \cup B^{*s}(I) \subseteq (K_1 \cup B)^{*s}(I)$. Let $x \in (K_1 \cup B)^{*s}(I)$.

Then, for each $G \in SO(X_1,x)$, $(G \cap K_1) \cup (G \cap B) = G \cap (K_1 \cup B) \in/ I$. Therefore,
$G \cap K_1 \in/ I$ or $G \cap B$ /∈ I. This implies that $x \in K_1^{*s}(I)$ or $x \in B^{*s}(I)$, that is
$x \in K_1^{*s} \cup B^{*s}$. Therefore, there are $(K_1 \cup B)^{*s}(I) \subset K_1^{*s}(I) \cup B^{*s}(I)$. Consequently, we obtain $(K_1 \cup B)^{*s}(I) = K_1^{*s}(I) \cup B^{*s}(I)$.

(e) Let $x \in (K_1^{*s})^{*s}(I)$. Then, for each $G \in SO(X_1,x), G \cap K_1^{*s}(I) \notin I$ and hence $G \cap K_1^{*s}(I) \neq \varnothing$. Let $y \in G \cap K_1^{*s}(I)$. Then $G \in SO(X_1,y)$ and $y \in K_1^{*s}(I)$. Hence,
there are $G \cap K_1 \in/ I$ and $x \in {K_1}^{*s}(I)$. This shows that $(K_1^{*s})^{*s}(I)) \subseteq K_1^{*s}(I)$.

(f) Let $x \notin K_1^{*s}(I)$, then for some semi-open set G, there are $x \in G$, $G \cap K_1 \in I$. This implies $G \subset X \backslash K_1^{*s}(I)$, which means $X \backslash K_1^{*s}(I)$ is a semi-open set. Thus, $K_1^{*s}(I)$ is semi-closed set in X_1.

(g) Since $K_1 \backslash E \subset K_1$, by (c) $(K_1 \backslash E)^{*s}(I) \subset K_1^{*s}(I)$. By (d) and Proposition3.1,
$(K_1 \cup E)^{*s}(I) = K_1^{*s}(I) \cup E^{*s}(I) = K_1^{*s}(I) \cup \emptyset = K_1^{*s}(I)$. Therefore, we obtain $(K_1 \backslash E)^{*s}(I) \subset K_1^{*s}(I) = (K_1 \cup E)^{*s}(I)$.

(h) Since $K_1 = (A \backslash B) \cup (B \cap K_1)$, by (d) there are $(K_1)^{*s}(I) = (K_1 \backslash B)^{*s}(I) \cup (B \cap K_1)^{*s}(I)$ and hence
$K_1^{*s}(I) \backslash B^{*s}(I) = K_1^{*s}(I) \cap (X_1 \backslash B^{*s}(I)) = [(K_1 \backslash B$ $(X \backslash B^{*s}(I))$
$= [(K_1 \backslash B)^{*s}(I) \cap (X \backslash B^{*s}(I))] \cup [(B \cap K)^{*s}(I) \cap (X \backslash B^{*s}(I))]$
$= [(K_1 \backslash B)^{*s}(I) \backslash B^{*s}(I)] \cup \emptyset \subset (K_1 \backslash B)^{*s}(I)$.

(i) Assume that G be an open set and $x \in G \cap K_1^{*s}(I)$. Then $x \in G$ and $x \in K_1^{*s}(I)$.
Let $H \in SO(X_1,x)$. Then $H \cap G \in SO(X_1,x)$ and $H \cap (G \cap K_1) = (H \cap G) \cap K_1 \in/ I$. This shows that $x \in (G \cap K_1)^{*s}(I)$ and hence we obtain $G \cap K_1^{*s}(I) \subset (G \cap K_1)^{*s}(I)$.
Moreover, $G \cap K_1^{*s}(I) \subset G \cap (G \cap K_1)^{*s}(I)$ and by (c) $K_1^{*s}(I) \supset (G \cap K_1)^{*s}(I)$ and $K_1^{*s}(I) \supset G \cap (G \cap K_1)^{*s}(I)$. Therefore, $G \cap K_1^{*s}(I) = G \cap (G \cap K_1)^{*s}(I)$. (j), and (k) see [9,11].

THEOREM 3.1

Let (X_1,τ) be a topological structure with two K ideals **I, J** on X_1 and A be a subset of X_1. Then, the next properties hold:
(1) If $I \subseteq J$, then $K_1^{*s}(J) \subseteq K_1^{*s}(I)$;
(2) $K_1^{*s}(I \cap J) = K_1^{*s}(I) \cup K_1^{*s}(J)$.

PROOF.

(1) Let $I \subset J$ and $x \in K_1^{*s}(J)$. Then $K_1 \cap G$ /∈ J for each $G \in SO(X_1,x)$. Since $I \subset J$, $K_1 \cap G$ /∈ I and hence $x \in K_1^{*s}(I)$. Therefore, there are $K_1^{*s}(J) \subset K_1^{*s}(I)$.
(2)Since $I \cap J \subset I$ and $I \cap J \subset J$, then by (1), $K_1^{*s}(I) \subset K_1^{*s}(I \cap J)$ and $K_1^{*s}(J) \subset K_1^{*s}(I \cap J)$. Therefore, we obtain $K_1^{*s}(I) \cup K_1^{*s}(J) \subset K_1^{*s}(I \cap J)$............(i)
now, let $x \in K_1^{*s}(I \cap J)$. Then, for each $G \in SO(X_1,x)$, $G \cap A$ /∈ $I \cap J$ and hence $G \cap K_1 \in/ I$ or $G \cap K_1 \in/ J$. This shows that $x \in K_1^{*s}(I)$ or $x \in K_1^{*s}(J)$. Therefore, there are $x \in K_1^{*s}(I) \cup K_1^{*s}(J)$. This shows that $K_1^{*s}(I \cap J) \subset {K_1}^{*s}(I) \cup K_1^{*s}(J)$ ....(ii)
From (i), (ii) , we obtain $K_1^{*s}(I \cap J) = {K_1}^{*s}(I) \cup K_1^{*s}(J)$.

4. SOME CHARACTERIZATIONS OF NANO SEMI- LOCAL FUNCTIONS

the nano local function of a nI with regard to when it is defined on P (X) as:
$K_1^{*}(\tau, I) = X_1 \backslash \cup \{G \in \tau : G \cap K_1 \in I\} (K_1 \subseteq X_1)$ [2] or equivalently,
$K_1^{*}(\tau, I) = \{x \in X_1 : \forall G_x \in \tau_x, G_x \cap K_1 \notin I\}$, where $\tau_x = \{G \in \tau : x \in G\}$.

PROPOSITION 4.1.

The following two properties are equivalents for any n$I\tau$ *structure* (X_1, τ, I) (P_1) $K_1 \in I$ *iff* $K_1^{*}(I) = \varnothing$ *iff* $K_1 \cap K_1^{*}(I) = \varnothing$.
(P₂) If $\{K_{1\propto} : \propto \in \nabla\}$ *is a family of sets belonging to I and each* $K_{1\propto}$ *is an open set in the subspace* $\cup\{K_{1\propto} : \propto \in \nabla\}$, *then* $\cup\{K_{1\propto} : \propto \in \nabla\} \in I$.

REMARK 2.1.

If an nII satisfies (P_1), then $\beta(I) = \tau^{*}(I)$ (where $\beta(I) = \{G \backslash E : G \in \tau, E \in I\}$ is a base of the nτ $\tau^{*}(I)$ in X_1). [8], this equality is not necessary, as shown by the following example due to Ewert [8].

DEFINITION 4.2.

The nano semi-local function [14] of a nI **I** with respect to τ is defined on P(X) as: -

$K_1^{*s}(I,\tau)$ = {$x\epsilon X$: $G^T K_1 \notin I$ for each $G\epsilon SO(X_1,\tau)$}. writing simply $K_1^{*s}(I)$ for $K_1^{*s}(I,\tau)$.

LEMMA 4.3.

If a nI has the property (P_1), then $K_1 \cup K_1^{*s}(I) = sCl^*(K_1)$ for each set $A \subseteq X_1$.

PROPOSITION 4.4.

Let (X_1,τ,I) be an nIτ and $K_1 \subseteq X_1$ thus.

(1) $K_1^{*s}(I) \subseteq K_1^*(I), \forall K_1 \subseteq X_1$.

(2) $K_1^{*s}(I) = K_1^*(I)$ if $\tau = SO(X_1,\tau)\}$.

(3) If $K_1 \in I$, then $K_1^*(I) = \varnothing$.

(4) In general, neither $\beta \subseteq K_1^{*s}(I)$ nor $K_1^{*s}(I) \subseteq K_1$.

REMARK 4.5.

If a nII in a nτstructure (X_1,τ) satisfies property (P_1), then ∀ $K_1 \subseteq X_1$, the sets $Cl^*(K_1)\backslash sCl^*(K_1)$ and $sInt^*(K_1)\backslash Int^*(K_1)$ are nowhere dense in (X_1,τ).

PROPOSITION 4.6.

Let (X_1, τ,I) be a nIτ structure and $K_1,B \subseteq X_1$. The following characteristics hold for the nano semi-local function::

(a) $K_1^{*s}(I) = sCl(K_1^{*s}(I)) \subseteq sCl(K_1)$;

(b) $K_1^{*s}(I) \subset K_1^*(I)$;

(c) If $K_1 \subseteq B$, then $K_1^{*s}(I) \subseteq B^{*s}(I)$;

(d) $(K_1 \cup B)^{*s}(I) = K_1^{*s}(I) \cup B^{*s}(I)$;

(e) $(K_1^{*s})^{*s}(I) \subseteq K_1^{*s}(I)$;

(f) $K_1^{*s}(I)$ is a semi-closed set in X_1.

(g) If $E \in I$, then $(K_1\backslash E)^{*s}(I) \subset K_1^{*s}(I) = (K_1 \cup E)^{*s}(I)$;

(h) $(K_1)^{*s}(I)\backslash B^{*s}(I) = (K_1\backslash B)^{*s}(I)\backslash B^{*s}(I) \subset (K_1\backslash B)^{*s}(I)$;

(i) If $G \in \tau$, then $G \cap K_1^*(I) = G \cap (G \cap K_1)^{*s}(I) \subseteq (G \cap K_1)^{*}$;

(j) If $I = \{\emptyset\}$, then $K_1^{*s}(I) = sCl(K_1)$;

(k) If $I = P(X_1)$, then $K_1^*(I) = \varnothing$, gives $K_1^{*s}(I) = \varnothing$, for each $K_1 \subseteq X_1$;

THEOREM 4.7.

Let (X_1,τ) be a topological structure with two K nI I, J on X_1 and A be a subset of X_1. Then, the next properties hold:

(1) If $I \subseteq J$, then $K_1^{*s}(J) \subseteq K_1^{*s}(I)$;

(2) $K_1^{*s}(I \cap J) = K_1^{*s}(I) \cup K_1^{*s}(J)$.

5. CONCLUSIONS

The development of nano ideal topology in mathematical structures of nano approximations has attracted many experts in many fields. In this work we have investigated the concept of semi-local functions due to the first author in [14]. Also, we observed that semi local functions are not a Kuratowski closure operator. We established some properties and characterizations of nano semi local functions. We introduced a real-life application based on nano semi local mappings. We established the best path for accessing the vital places in our city. Through the future work, we will investigate more applications for nI- topology such as human blood circulation and provide a topological technique for getting the vigour of different biological solutions.

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