

RUBBER SHEET GEOMETRY IN DESIGN PROCESS

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A. Salam^{a,*}, M. Darwish^b, M. Shokry^b, M. A. Nasef^{b,*}

^a Department of Mathematics, Faculty of Science, Tanta University, Egypt

^b Department of Physics and Engineering Mathematics, Faculty of Engineering, Tanta University, Egypt

* corresponding author: M. A. Nasef (arafa_x_99@yahoo.com)

ABSTRACT. Topological Geometry is the rich field of mathematics that exists in many branches of engineering sciences. In addition, it is used in various real-life applications. In this paper, we concentrate on the publication of Möbius surface as a new model of Geometrical Topological Structure (GTS). Also, we show that there exist some models from rubber sheet geometry that can be applied to design processes.

KEYWORDS: Topological Geometry; Rubber sheet geometry; Design process; Möbius strip

1. INTRODUCTION

Topological knowledge plays an important role in computer graphics and image analysis. Consequently, Topology [1, 2, 8–10] is a branch of mathematics whose mathematical structures can be used for modelling phenomena of the real world. It also provides an approach for data analysis and knowledge discovery and has been studied intensively in several fields of social sciences, engineering technologies, physical sciences, and environmental sciences.

Moreover, topological geometry problems have close connections to important industrial applications. In this work, by using only one model from geometrical topological structure, we present a new model called Möbius surface [3,4,6,7,11,16,17] and its different geometrical applications in Architecture forms and design processes in many fields. Thus, the motivation of the present paper is to show that there are some models from rubber sheet geometry that can be applicable to design processes.

2. A BRIEF HISTORY OF THE MÖBIUS STRIPS

In 1858, Ferdinand M. Möbius and Johann Benedict Listing discovered a novel kind of rubber sheet geometry called Möbius Band (or Möbius strip), as shown in Fig. 1.

The Möbius strip Möbius band Möbius loop, or Möbius surface, is also called a twisted cylinder. This model is an example of a one-sided surface in the form of A single closed continuous curve with a twist.

- *Why is the Möbius Strip attributed to topology?*

For the topology searches in the general properties of shapes

2.1. HOW TO MAKE A MÖBIUS STRIP?

An obvious Strip can be created by taking a strip of paper, giving it an odd number of half-twists, and then taping the ends back together to form a loop. If you take a pencil and draw a line in the centre of the strip, you'll see that the line apparently runs along both sides of the loop, as illustrated in Fig. 2.

2.2. THE GENERAL PROPERTIES OF A RUBBER SHEET GEOMETRY

One of the most important general characteristics of the organised and complex topological spaces for shape generation and transformation is the following: The rubber sheet geometry has several interesting properties that can be interpreted in architecture and other fields:

- Time, movement, and changing properties of the spaces.
- Merging between inside and outside and realising the idea (theory) of visual fluidity inside and outside space.
- Moving from standard units to digital genes and abandoning standardisation systems.
- Increasing the transparency of the architectural formation elements.
- The blurring of boundaries between inside and outside.
- Freedom of creativity in implementation processes from spatial determinants

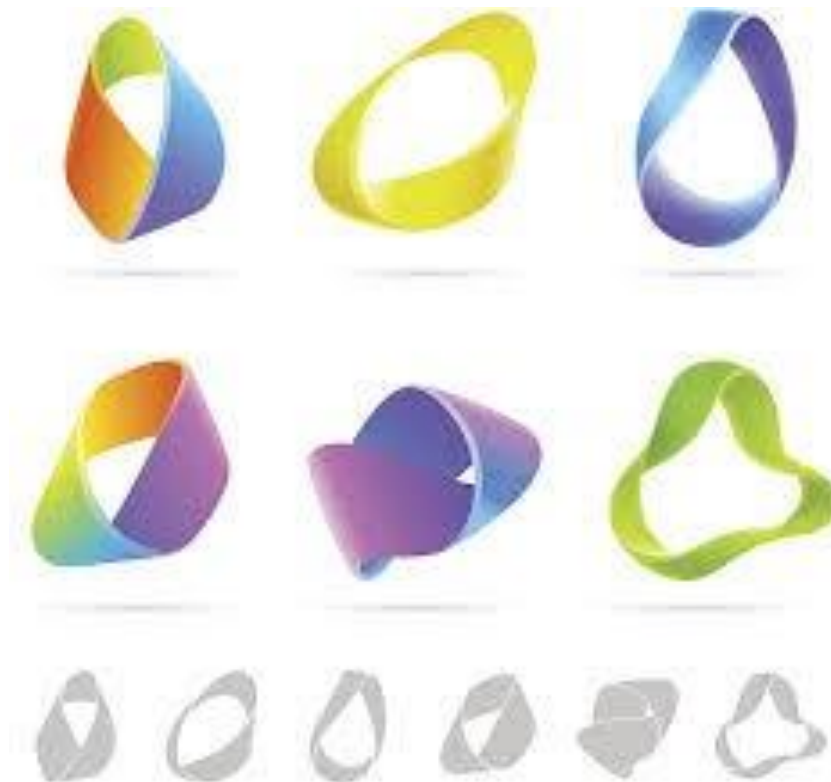


Fig. 1. Möbius Geometry Shapes.

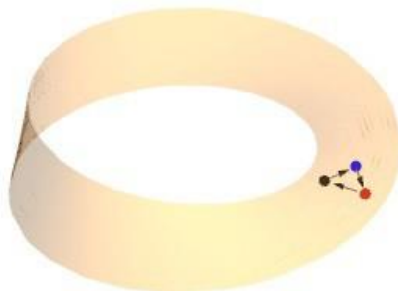


Fig. 2. Möbius Strip

3. REAL LIFE APPLICATIONS OF A MÖBIUS SURFACE

3.1. NATIONAL LIBRARY IN ASTANA, KAZAKHSTAN

This library was designed by Thomas Christoffersen using the Möbius strip. The design is an infinite circular loop and a double loop surface that covers the entire building, surrounded by light and air on all sides. More advantages can be considered as follows:

- In the middle of the building, there is a wide courtyard to receive light through a heavenly dome that allows clear guidance of sunlight so that it fills the shelves of the library.
- The connected surfaces that envelop the library were designed in a style and technique different from traditional architecture, and this is by means of topological engineering, which allows the stretching

and twisting of any part of the building.

- The library interface was designed as a coexistence between Urbanization and nature, to be open to the outside, to draw a wonderful aesthetic painting in which the picturesque landscapes shie with the surrounding environment.

Moreover, the thermal exposure on the building envelope was calculated using state-of-the-art technology and simulation capacity. Due to the warping and twisting geometry, the thermal imprint on the façade is continually varying in intensity. The thermal map from blue to red reveals which zones do and do not need shading. By translating the climatic information into a façade pattern of varying openness, a form of ecological ornament that regulates the solar impact according to thermal requirements is satisfied. The use of topological geometry in the external and internal architectural design of the library and the twisting of the building is noted by the Möbius band in the design, as presented in Fig. 3



Fig. 3. Möbius Strip, National Library in Astana, Kazakhstan.

3.2. LITERATURE AND ART CENTER OF TAICHUNG CITY, TAIWAN

This centre is from design (Vincent Callebaut Architectures). The design was based on Möbius Strips, which wrap around in space between binary and triple space (D^2 , D^3), creating an infinite surface out of it. The idea was based on a series of triangles that gradually rotate around an ellipse, where the walls are semi-circular.

This building allows the integration of the public space in the centre of the building by twisting the structure, and the structure is raised above the ground to allow access to the internal space of the building. The design was based on Möbius strips that wrap somewhere between 2D and 3D, creating an infinite surface, as shown in Fig. 4.

3.3. URBAN NIGHT CLUB AND CYBERPUNK CULTURE, TOKYO

This club was designed by both professors (Jiannan Liu and Tingwei Xu) from the University of Pennsylvania. Figure 5 shows that the design focuses on affective and dynamic formations for this nightclub in space. The project's goal was to build a connected space with different transformation and surface transition that extends from the inside to the outside, where the topological surfaces are transformed from one surface to another, which leads to the user's sense of variations in the public space with the connectivity and continuity of the space visually from interiors to exteriors.

It is one of the most creative and beautiful architectural engineering works in terms of space connected with various transformations.

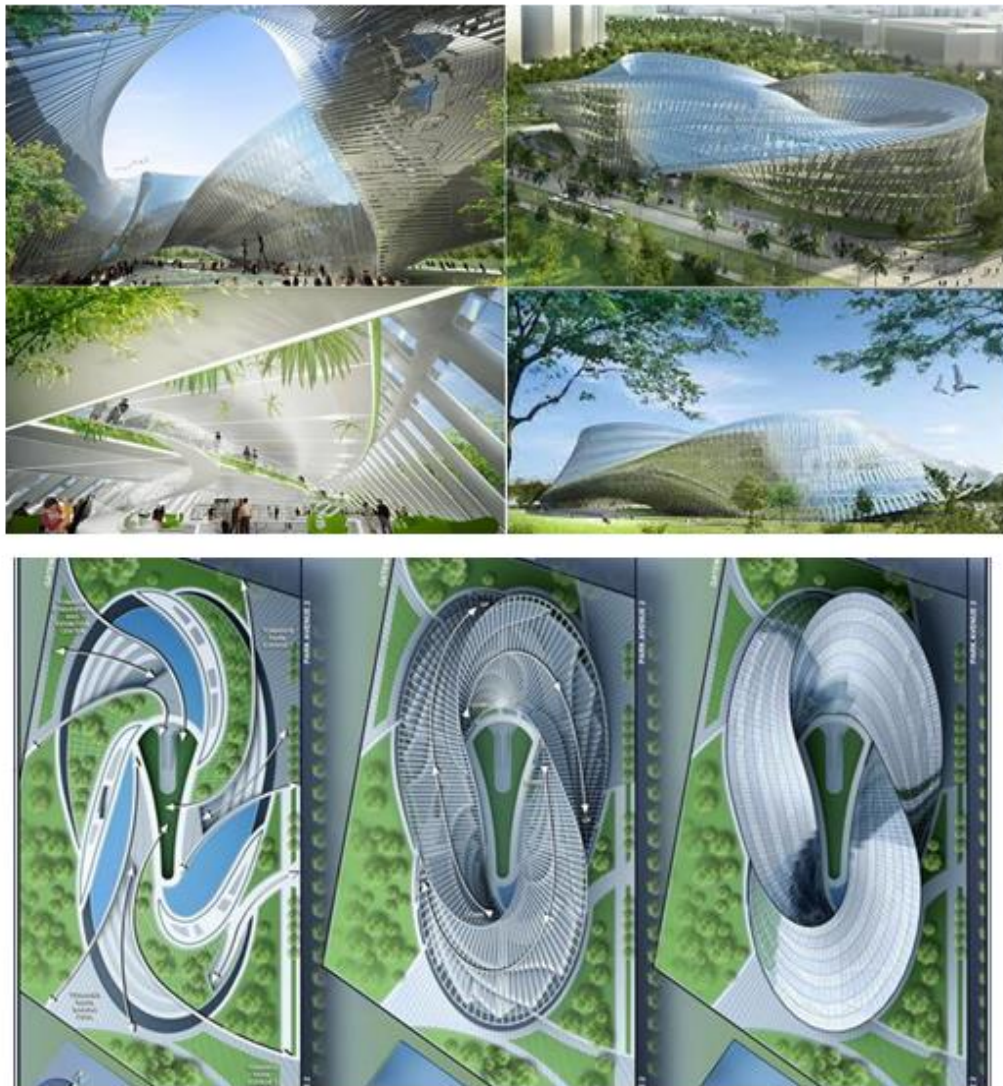


Fig. 4. Vincent Callebaut Architectures: Möbius Strip Building Brings Loops And Bling to Taiwan



Fig. 5. Internal shots of the inner space are characterised by dynamic and impactful formations, which are connected and continued topological spaces

3.4. OTHER APPLICATIONS POLICE GIANT MÖBIUS

Giant Möbius strips have been used in many industries, such as:

1. Möbius strips are used in the manufacture of continuous loop recording tapes in order to double the playback period.
2. Used in typewriter ribbons and printer ribbons.
3. Möbius tapes are used in the manufacture of conveyor belts, as their life span is twice that of other belts.
4. In addition to the above, the Möbius strips are a

source of inspiration for sculpture and graphic art.

5. Möbius strips are used in science fiction when writing novels, literary works, and movies.
6. Möbius strips are also used in the jewellery industry. The Möbius strip has been tailored to various artistic and cultural products. Paintings have displayed Möbius shapes, as have earrings, necklaces, and other pieces of jewellery, as shown in Fig .6. Furthermore, the green, three-armed universal sign for recycling also composes the Möbius band. There's a depth to the image that reminds you to reduce, reuse and recycle.



Fig. 6. Giant Möbius strip

4. SUMMARY AND DISCUSSIONS

The Geometrical Topological structures of the Möbius surface have great potential as an architectural form that is difficult to visualise and investigate without the aid of digital technologies. This work demonstrates that it is possible to develop a building that is a pure translation of the model of the Möbius strip, and it furthers a current trend in architectural forms being developed from mathematical concepts beyond our inspiration. The concept of the Möbius Band also appears to be used in many fields, such as Möbius House, Möbius car, Möbius in art, Möbius seating, and Möbius Museum [5, 14, 15]. The Möbius strip, since its discovery, has captured the imaginations of mathematicians and engineers everywhere, and it continues to fascinate modern generations with its non-orientability and one-sidedness.

5. CONCLUSION

The utilisation of rubber sheet techniques exhibits significant potential as an innovative and adaptable supplement to the first phases of the design process. However, the use of precise integration with well-organized Computer-Aided Design (CAD) and

Computer-Aided Engineering (CAE) technologies is imperative in order to effectively transform inventive ideas into concrete items and systems. Further investigation is required to explore hybrid processes that effectively utilise the advantages of both flexible form discovery and sturdy engineering systems.

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