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IRRESOLUTE MAPS IN TOPOLOGICAL ORDERED SPACES

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V.V.S.Ramachandram[a,*], D.Nagapurnima[b]

[a] *Professor in Mathematics, Department of H&BS, ISTS Engineering College for Women, Rajahmundry, Andhrapradesh*
[b] *Professor in Mathematics, Department of H&BS, RIET Engineering College, Rajahmundry, Andhrapradesh*
[*] corresponding author: V.V.S.Ramachandram (vvsrch@gmail.com)

ABSTRACT. The notion of semi-generalized closed set (sg-closed set) was first introduced by Bhattacharya and Lahiri. The notion of semi-open set was introduce by N.Levine **[5].**The complement of a semi open set is a semi closed set. These sets were also considered by various authors and studied their properties. In 1965, L.Nachbin **[4]** introduced topological ordered spaces. Later some authors extended these concepts to topological ordered spaces.A topological space along with a partial order is named as a topological ordered space. Some Authors introduced and studied the notion of irresolute maps in topological spaces **[9]**. These types of maps can be defined on topological ordered spaces. In this paper, irresolute maps were defined on topological ordered spaces via increasing, decreasing and balanced sets. Some properties were established with examples.

KEYWORDS: Irresolute map; Contra irresolute map; Topological ordered set; Increasing set; Decreasing set; Balanced set.

AMS subject classification: 54C55, 54A05

1. Introduction

L.Nachbin **[4]** introduced the notion of topological ordered spaces. A topological ordered space is a triple $(X,\tau,\leq)$ where X is a nonempty set, τ is a topology and $\leq$ is a partial order. For $x\in X$, define $[x,\rightarrow]=\{y\in X/x\leq y\}$ and $[\leftarrow,x]=\{y\in X/y\leq x\}$.For a subset A, define $i[A]=\cup_{a\in A}[a,\rightarrow]$ and $d[A]=\cup_{a\in A}[\leftarrow,a]$.Then, A is increasing if $A=i[A]$ and decreasing if $A=d[A]$. A subset is balanced if it is both increasing and decreasing.

2. Preliminaries

The following definitions are useful in the present work.

Definition 2.1. [5]

A subset A of a topological space (X,τ) is a semi-open set if $A\subseteq \mathrm{cl}(\mathrm{int}(A))$ and a semi closed set if $\mathrm{int}(\mathrm{cl}(A))\subseteq A$.

Definition 2.2. [1]

A subset $A$of a topological space (X,τ) is a semigeneralized closed set (sg-closed) if $\mathrm{scl}(A)\subseteq U$ whenever $A\subseteq \mathrm{U}$ and U is semi-open in (X,τ).

Definition 2.3.[4]

A subset $A$of a topological ordered space $(X,\tau,\leq)$ is an increasing semi closed set (i-semi closed) if A is a semi closed set and an increasing set.

Definition 2.4. [4]

A subset $A$of a topological ordered space $(X,\tau,\leq)$ is a decreasing semi closed set (d-semi closed) if A $A$is a semi closed set and a decreasing set.

Definition 2.5. [4]

A subset A of a topological ordered space $(X,\tau,\leq)$ is a balanced semi closed set (b-semi closed) if A is a semi closed set and a balanced set.

Definition 2.6. [7]

A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is an irresolute map if $f^{-1}(S)$ is a semi closed set in X for a semi closed subset S of Y.

3. Increasing Irresolute Maps

In this section we introduce the concept of irresoluteness in topological ordered spaces using increasing sets. We define the following map.

Definition 3.1.

A mapping $f:(X,\tau,\leq_1)\rightarrow(Y,\sigma,\leq_2)$is an Increasing irresolute map (I_i map) if $f^{-1}(S)$ is an i-semi closed set in X for any i- semi closed subset S of Y.

EXAMPLE 3.2.

Let $X = \{a,b,c\}$,$\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and $\leq_9 = \{(a,a),(b,b),(c,c),(a,c)\}$. Then, $(X,\tau_1,\leq_9)$ is a TOS. The i-semi closed sets are $\emptyset$,X, $\{b\},\{c\}$, $\{b,c\},\{a,c\}$. Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and $\leq_{10} = \{(a,a),(b,b),(c,c),(c,a),(b,c),(b,a)\}$. Then,$(X,\tau_1,\leq_{10})$ is a TOS. The i-semi closed sets are $\emptyset$,X,$\{a\},\{a,c\}$. Define a map $f:(X,\tau_1,\leq_9) \rightarrow (X,\tau_1,\leq_{10})$ by $f(a) = b, f(b) = a, f(c) = c$. It is clear that, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(X) =$ X, $f^{-1}(\{a\}) = \{b\}$ and $f^{-1}(\{a,c\}) = \{b,c\}$. This shows that the inverse image of every i-semi closed set in the space $(X,\tau_1,\leq_{10})$is an i-semi closed set in the space $(X,\tau_1,\leq_9)$. Hence, the map f is a I_i map.

4. DECREASING IRRESOLUTE MAPS

In this section we introduce the concept of irresoluteness in topological ordered spaces using decreasing sets. We define the following map.

DEFINITION 4.1.

A mapping $f:(X,\tau,\leq_1) \rightarrow (Y,\sigma,\leq_2)$is a decreasing irresolute map (D_i map) if $f^{-1}(S)$ is a d-semi closed set in X for any d-semi closed subset S of Y.

EXAMPLE 4.2.

Let$X = \{a,b,c\}$,$\tau_8 = \{\emptyset, X, \{a,b\}\}$ and $\leq_5 = \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$.Then, $(X,\tau_8,\leq_5)$ is a TOS. The d-semi closed sets are $\emptyset, X$.Let $X = \{a,b,c\}$, $\tau_8 = \{\emptyset, X, \{a,b\}\}$ and $\leq_6 = \{(a,a),(b,b),(c,c),(b,a),(b,c),(a,c)\}$.Then, $(X,\tau_8,\leq_6)$ is a TOS. The d-semi closed sets are $\emptyset, X$. Define a map $f:(X,\tau_8,\leq_5) \rightarrow (X,\tau_8,\leq_6)$ by$f(a) = b, f(b) = a, f(c) = c$. It is clear that, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(X) = X$.This shows that the inverse image of every d-semi closed set in the space $(X,\tau_8,\leq_6)$is a d-semi closed set in the space$(X,\tau_8,\leq_5)$.Hence, the map f is a $D_i$map.

5. BALANCED IRRESOLUTE MAPS

In this section we introduce the concept of irresoluteness in topological ordered spaces using balanced sets. We define the following map.

DEFINITION 5.1.

A mapping $f:(X,\tau,\leq_1) \rightarrow (Y,\sigma,\leq_2)$is a balanced irresolute map (B_i map) if $f^{-1}(S)$is a b-semi closed set in X for any b-semi closed subset S of Y.

EXAMPLE 5.2.

Let $X = \{a,b,c\}$, $\tau_6 = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ and $\leq_8 = \{(a,a),(b,b),(c,c)\}$. Then, $(X,\tau_6,\leq_8)$ is a TOS. The b-semi closed sets are $\emptyset$,X,$\{b,c\},\{b\},\{c\},\{a,c\}$. Let $X = \{a,b,c\}$,$\tau_6 = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ and $\leq_9 = \{(a,a),(b,b),(c,c),(a,c)\}$. Then, $(X,\tau_6,\leq_9)$ is a TOS. The b-semi closed sets are $\emptyset$, X, $\{b\},\{a,c\}$. Define a map $f:(X,\tau_8,\leq_5) \rightarrow (X,\tau_8,\leq_6)$ by $f(a) = b, f(b) = a$, $f(c) = c$. It is clear that, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(X) = X$,$\boldsymbol{f^{-1}(\{b\}) = \{b\}}$,$\boldsymbol{f^{-1}(\{a,c\}) = \{a,c\}}$. This shows that the inverse image of every b-semi closed set in the space $(X,\tau_6,\leq_9)$is a b-semi closed set in the space$(X,\tau_6,\leq_8)$. Hence, the map$f$ is a $B_i$map.

6. INCREASING, DECREASING AND BALANCED SEMI OPEN MAPS

In this section, we define new maps namely increasing semi open map (i-semi open map), decreasing semi open map (d-semi open map) and balanced semi open map (b-semi open map).

DEFINITION 6.1: I-SEMI OPEN MAP:

A mapping $f:(X,\tau,\leq_1) \rightarrow (Y,\sigma,\leq_2)$is an increasing semi open map (i-semi open map)

if the image of an i-open set in X is an i-semi open set in Y.

EXAMPLE 6.2:

Let $X = \{a,b,c\}$, $\tau_2 = \{\emptyset, X, \{a\}\}$ and $\leq_6 = \{(a,a),(b,b),(c,c),(b,a),(b,c),(a,c)\}$Then, $(X,\tau_2,\leq_6)$ is a TOS. The i-open sets are $\emptyset, X$.Let $X = \{a,b,c\}$,$\tau_2 = \{\emptyset, X, \{a\}\}$ and p$\leq_5 = \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$.Then, $(X,\tau_2,\leq_6)$ is a TOS. The i-semi open sets are $\emptyset, X, \{a\}, \{a,b\}$.Define a map $f:(X,\tau_2,\leq_6) \rightarrow (X,\tau_2,\leq_5)$ by$f(a) = a, f(b) = c, f(c) = b$. It is clear that, $f(\emptyset) = \emptyset, f(X) = X$.This shows that the image of every i-open set in the space $(X,\tau_2,\leq_6)$is an i-semi open set in the space $(X,\tau_2,\leq_5)$. Hence, the map f is an i-semi open map.

DEFINITION 6.3: D-SEMI OPEN MAP:

A mapping $f:(X,\tau,\leq_1) \rightarrow (Y,\sigma,\leq_2)$is a decreasing semi open map (d-semi open map)

if the image of a d-open set in X is a d-semi open set in Y.

EXAMPLE 6.4:

Let $X = \{a,b,c\}$,$\tau_2 = \{\emptyset, X, \{a\}\}$and $\leq_6 = \{(a,a),(b,b),(c,c),(b,a),(b,c),(a,c)\}$Then, $(X,\tau_2,\leq_6)$ is a TOS. The d-open sets are $\emptyset, X$. Let $X = \{a,b,c\}$,$\tau_2 = \{\emptyset, X, \{a\}\}$ and $\leq_5 = \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$.Then, $(X,\tau_2,\leq_5)$is a TOS. The d-semi open sets are $\emptyset$, X, $\{a,c\}$. Define a map $f:(X,\tau_2,\leq_6) \rightarrow (X,\tau_2,\leq_5)byf(a) = a, f(b) = c, f(c) = b$. It is clear that, $f(\emptyset) = \emptyset$, $f(X) = X$.This shows that the image of every d-open set in the space $(X,\tau_2,\leq_6)$is a d-semi open set in the space$(X,\tau_2,\leq_5)$. Hence, the map$f$ is a d-semi open map.

DEFINITION 6.5: B-SEMI OPEN MAP:

A mapping $f:(X,\tau,\leq_1) \to (Y,\sigma,\leq_2)$is a balanced semi open map (b-semi open map)

if the image of a b-open set in X is a b-semi open set in Y.

EXAMPLE 6.6:

Let $X = \{a,b,c\}$,$\tau_2 = \{\emptyset, X, \{a\}\}$and $\leq_6= \{(a,a),(b,b),(c,c),(b,a),(b,c),(a,c)\}$. Then,$(X,\tau_2,\leq_6)$ is a TOS. The b-open sets are $\emptyset, X$. Let $X = \{a,b,c\}$,$\tau_2 = \{\emptyset, X, \{a\}\}$ and $\leq_5= \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$.Then,$(X,\tau_2,\leq_5)$is a TOS. The b-semi open sets are$\emptyset, X$. Define $f:(X,\tau_2,\leq_6) \to (X,\tau_2,\leq_5)$ by$f(a) = a, f(b) = c, f(c) = b$. It is clear that $f(\emptyset) = \emptyset$, $f(X) = X$. So, the map $\boldsymbol{f}$ is a b-semi open map.

7. RESULTS ON IRRESOLUTE MAPS

THEOREM 7.1:

If $f:X \to Y$ and g :Y $\to Z$ are two I_i maps, then $gof:X \to Z$is also an I_i map.

PROOF:

Let $\boldsymbol{S}$ be an i-semi closed subset of Z.Since,g :Y $\to Z$ is an I_i map, $g^{-1}(S)$is an i-semi closed subset of Y. Since, $f:X \to Y$ is an I_i map, $f^{-1}(g^{-1}(S))$ is an i-semi closed subset of X. But, $f^{-1}(g^{-1}(S)) = (gof)^{-1}(S)$ and hence, $gof:X \to Z$ is an I_i map.

THEOREM 7.2:

If $f:X \to Y$ and g :Y $\to Z$ be two D_i maps, then $gof:X \to Z$is also a D_i map.

PROOF: TRIVIAL.

THEOREM 7.3:

If $f:X \to Y$ and g :Y $\to Z$ be two B_i maps, then $gof:X \to Z$is also a B_i map.

Proof: Trivial.

THEOREM 7.4:

Suppose $f:X \to Y$ and g :Y $\to Z$ are two maps such that g is an I_i map and $gof:X \to Z$is an i-semi open map, then $f:X \to Y$is an i-semi open map.

PROOF:

Let $\boldsymbol{S}$ be an i-open subset of X.Since, $gof:X \to Z$ is an i-semi open map, $gof(S)$ is an i-semi open subset of Z.But,$gof(S) = g(f(S))$ and g :Y $\to Z$ is an $I_i$map, we have $g^{-1}[g(f(S))]$is an i-semi open subset of Y.Since,$g^{-1}[g(f(S))] = f(S)$, the mapping $f:X \to Y$ is an i-semi open map.

THEOREM 7.5:

Suppose $f:X \to Y$ and g :Y $\to Z$ are two maps such that g is a D_i map and $gof:X \to Z$is a d-semi open map, then $f:X \to Y$is a d-semi open map.

PROOF: TRIVIAL.

THEOREM 7.6:

Suppose $f:X \to Y$ and g :Y $\to Z$ are two maps such that g is a B_i map and $gof:X \to Z$is a b-semi open map, then $f:X \to Y$is a b-semi open map.

PROOF: TRIVIAL.

8. CONTRA IRRESOLUTE MAPS IN TOPOLOGICAL ORDERED SPACES

In this section we introduce the concept of contra irresoluteness[9] in a topological ordered space by using increasing, decreasing and balanced sets.

DEFINITION 8.1:

A mapping $f:(X,\tau,\leq_1) \to (Y,\sigma,\leq_2)$is a contra increasing irresolute map (${I_i}^c$map) if $f^{-1}(O)$is an i-semi closed set in X for each i-semi open set O in Y.

EXAMPLE 8.2:

Let $X = \{a,b,c\}$, $\tau_3 = \{\emptyset, X, \{a\}, \{b,c\}\}$ and $\leq_8= \{(a,a),(b,b),(c,c)\}$.Then, $(X,\tau_3,\leq_8)$is a TOS. The i-semi closed sets are $\emptyset, X, \{a\}, \{b,c\}$.Let $X = \{a,b,c\}$, $\tau_3 = \{\emptyset, X, \{a\}, \{b,c\}\}$ and $\leq_7= \{(a,a),(b,b),(c,c),(b,a)\}$.Then, $(X,\tau_3,\leq_7)$ is a TOS. The i-semi open sets are $\emptyset, X, \{b,c\}$. Define a map $f:(X,\tau_3,\leq_8) \to (X,\tau_3,\leq_7)$ by $f(a) = a, f(b) = c, f(c) = b$. It is clear that, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(X) = X$, $\boldsymbol{f^{-1}(\{b,c\}) = \{b,c\}}$This shows that the inverse image of every i-semi open set in the space $(X,\tau_3,\leq_7)$is a i-semi closed set in the space $(X,\tau_3,\leq_8)$. Hence, the map$f$ is a ${I_i}^c$map.

DEFINITION 8.3:

A mapping $f:(X,\tau,\leq_1) \to (Y,\sigma,\leq_2)$is a contra decreasing irresolute map (${I_d}^c$map) if $f^{-1}(O)$is a d-semi closed set in X for each d-semi open set O in Y.

EXAMPLE 8.4:

Let $X = \{a,b,c\}$,$\tau_3 = \{\emptyset, X, \{a\}, \{b,c\}\}and\leq_8= \{(a,a),(b,b),(c,c)\}$.Then, $(X,\tau_3,\leq_8)$ is a TOS. The i-semi closed sets are $\emptyset, X, \{a\}, \{b,c\}$.Let $X = \{a,b,c\}$,$\tau_3 = \{\emptyset, X, \{a\}, \{b,c\}\}$and $\leq_7= \{(a,a),(b,b),(c,c),(b,a)\}$.Then,$(X,\tau_3,\leq_7)$ is a TOS. The d-semi open sets are $\emptyset, X, \{a\}$. Define a map $f:(X,\tau_3,\leq_8) \to (X,\tau_3,\leq_7)$ by $f(a) = a, f(b) = c, f(c) = b$.It is clear that, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(X) = X$, $f^{-1}(\{a\}) = \{a\}$.This shows that the inverse image of every d-semi open set in the space $(X,\tau_3,\leq_7)$ is a d-semi closed set in the space$(X,\tau_3,\leq_8)$. Hence, the map$f$ is a ${I_d}^c$map.

DEFINITION 8.5:

A mapping $f:(X,\tau,\leq_1) \to (Y,\sigma,\leq_2)$is a contra balanced irresolute map ($I_b{}^c$map) if $f^{-1}(O)$is a b-semi closed set in X for each b-semi open set O in Y.

EXAMPLE 8.6:

Let $X = \{a,b,c\}$,$\tau_3 = \{\emptyset, X, \{a\}, \{b,c\}\}$ and $\leq_8 = \{(a,a),(b,b),(c,c)\}$.Then, $(X,\tau_3,\leq_8)$ is a TOS. The b-semi closed sets are $\emptyset, X, \{a\}, \{b,c\}$.Let $X = \{a,b,c\}$, $\tau_3 = \{\emptyset, X, \{a\}, \{b,c\}\}$ and $\leq_7 = \{(a,a),(b,b),(c,c),(b,a)\}$.Then,$(X,\tau_3,\leq_7)$is a TOS. The b-semi open sets are $\emptyset, X$.Define a map $f:(X,\tau_3,\leq_8) \to (X,\tau_3,\leq_7)$ by$f(a) = b, f(b) = c, f(c) = a$. It is clear that, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(X) = X$.This shows that the inverse image of every b-semi open set in the space $(X,\tau_3,\leq_7)$is a b-semi closed set in the space$(X,\tau_3,\leq_8)$. Hence, the map$f$ is a $I_b{}^c$map.

9. RESULTS ON CONTRA IRRESOLUTE MAPS.

THEOREM 9.1:

If $f:X \to Y$ is an $I_i{}^c$ map and $g:Y \to Z$ is an I_i map, then $gof:X \to Z$is an $I_i{}^c$ map.

PROOF:

Let S be an i-semi open subset of Z.Since,$g:Y \to Z$ is an I_i map, $g^{-1}(S)$is an i-semi open subset of Y. Since, $f:X \to Y$ is an $I_i{}^c$ map, $f^{-1}(g^{-1}(S))$ is an i-semi closed subset of X. But, $f^{-1}(g^{-1}(S)) = (gof)^{-1}(S)$ and hence, $gof:X \to Z$ is an $I_i{}^c$ map.

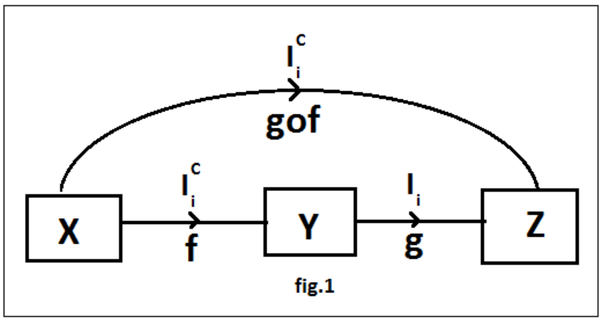

fig.1

THEOREM 9.2:

If $f:X \to Y$ is an $I_d{}^c$ map and $g:Y \to Z$ is an I_d map, then $gof:X \to Z$is an $I_d{}^c$ map.

PROOF: TRIVIAL.

THEOREM 9.3:

If $f:X \to Y$ is an $I_b{}^c$ map and $g:Y \to Z$ is an I_b map, then $gof:X \to Z$is an $I_b{}^c$ map.

PROOF: TRIVIAL.

THEOREM 9.4:

Suppose $f:X \to Y$ is an I_i map and $g:Y \to Z$ is an $I_i{}^c$map. Then, $gof:X \to Z$ is an $I_i{}^c$map.

PROOF:

Let S be an i-semi open subset of Z.Since,$g:Y \to Z$ is an $I_i{}^c$ map, $g^{-1}(S)$is an i-semi closed subset of Y. Since, $f:X \to Y$ is an I_i map, $f^{-1}(g^{-1}(S))$ is an i-semi closed subset of X. But, $f^{-1}(g^{-1}(S)) = (gof)^{-1}(S)$ and hence, $gof:X \to Z$ is an $I_i{}^c$ map.

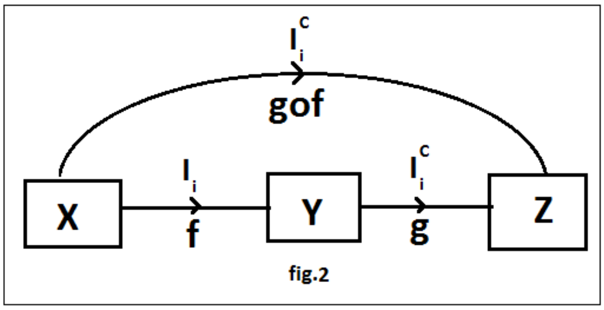

fig.2

THEOREM 9.5:

Suppose $f:X \to Y$ is an I_d map and $g:Y \to Z$ is an $I_d{}^c$map. Then, $gof:X \to Z$ is an $I_d{}^c$map.

PROOF: TRIVIAL.

THEOREM 9.6:

Suppose $f:X \to Y$ is an I_b map and $g:Y \to Z$ is an $I_b{}^c$map. Then, $gof:X \to Z$ is an $I_b{}^c$map.

PROOF: TRIVIAL.

10. CONCLUSIONS

In this paper the author extended the concept of irresolute maps in topological spaces to topological ordered spaces using increasing, decreasing and balanced sets. Some examples and basic relationships between the mappings were also discussed

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