Journal of Contemporary Technology and Applied Engineering (JCTAE), **2**(1), 2023, 12-18

DOI: 10.21608/JCTAE.2023.168503.1012 Published by: *Faculty of Engineering, Kafrelsheikh University*

NEARLY OPEN SETS IN INFRA TOPOLOGY AND RELATED INFRA CONTINUITY

Received: 12-10-2022
Accepted: 19-04-2023

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ABSTRACT. Infra topology has efficient tools of knowledge discovery. It has many applications in graph theory, fractals and game theory. One of the aims of this paper is to study some near infra open sets in infra topological spaces. Additionally, some results for near infra open and closed sets are studied. In fact, the union of an infra preclosed set and an infra α-closed set is an infra preclosed set. While, the intersection of an infra α-open set and an infra β-open set is infra β-open. Moreover, infra β-continuity is introduced and the relationships between some types of infra continuous functions is discussed. A comparison between near infra open sets and some other types of near open sets is investigated. Also, A comparison between near infra open sets and some other types of near open sets is obtained. Finally, we introduce two examples on infra topology to satisfy the relationships between infra β-continuity and some other types of infra continuity.

KEYWORDS: Infra topology, infra continuous functions, infra closed graphs.

1. INTRODUCTION

T. Husain [1] introduced the concept of infra topological spaces with respect to a subset X of a universe U. We study the relationships between some near infra open sets in infra topological spaces. In this paper we study the relationships between some weak forms of infra open sets in infra topological spaces. Also, we introduce the notion of infra β-continuity between infra topological spaces and we investigate several properties of these types of near infra continuity. Finally, we introduce some examples as an application in infra topological spaces. There are many applications of infra topology such as graph theory [17, 18, 19], fractals [20, 21] and game theory.

Infra topology is one of weak structures that can be extended in terms of simplicial complexes [25] and matroids with rough sets [24]. A new type of betweenness relation can be constructed [23]. It can be applied in nano topology as [26,27].

The main aim of this paper is establishing a new type of weak topological structures called near infra topological spaces, especially, infra β-open sets and infra α-open sets and main of its results are investigated. Additionally, many types of near infra continuous functions are introduced called infra β-continuous functions and their comparison with other types of near contionuous functions are studied.

2. BASIC CONCEPTS OF INFRA TOPOLOGY

Before entering in our work, we recall the following definitions which are useful in the sequel.

2.1. DEFINITION [1]

Let X be a nonempty set, a subfamily $i_X \subseteq P(X)$ is called an infra topology on X if it satisfies the following: (i) $\phi \in i_X$. (ii) If $G_\alpha \in i_X$, $1 \leq \alpha \leq n$, then $\bigcap_{\alpha=1}^{n} G_\alpha \in i_X$ or for $G_1, G_2 \in i_X$, $G_1 \cap G_2 \in i_X$.

Then the pair (X, i_X) is called an infra topological space. A set $V \in i_X$ is called an infra open set and its complement $X \backslash V$ is called an infra closed set.

2.2. PROPOSITION

Let (X, i_X) and (X, i'_X) be two infra topological spaces on X. Then:

(i) The intersection $i_X \cap i'_X$ is an infra topological space.

(ii) The union $i_X \cup i'_X$ is an infra topological space.

2.3. OBSERVATION

Statement 2 in Proposition 2.2 is not satisfied in general topological spaces.

2.4. DEFINITION [13]

Let (X, i_X) be an infra topological space. For a subset A of X, the infra closure of A and the infra interior of A are defined as follows:

(i) $Cl_i(A) = \bigcap \{F : A \subseteq F, X \backslash F \in i_X\}$ *and*

(ii) $Int_i(A) = \bigcup \{G : G \subseteq A, G \in i_X\}$.

2.5. Remark

Let X be a nonempty set, then: (i) If $i_X = \{X, \phi\}$, then (X, i_X) is called an indiscrete infra topological space or trivial infra topological space. (ii) If $i_X = P(X)$, then (X, i_X) is called a discrete infra topological space or maximal infra topological space.

2.6. Proposition

Every topological space is an infra topological space and the converse is not true as shown in the following example.

2.7. Example

Let $X = \{a, b, c\}$ and $i_X = \{X, \phi, \{a\}, \{b\}\}$. Then (X, i_X) is an infra topological space, but not topological space.

2.8. Definition

Let (X, i_X) be an infra topological space. Then $A \subseteq X$ is called: (1) Infra regular open if $A = Int_i(Cl_i(A))$. (2) Infra regular closed if $A = Cl_i(Int_i(A))$. (3) Infra semi-open if $A \subseteq Cl_i(Int_i(A))$. (4) Infra semi-closed if $Int_i(Cl_i(A)) \subseteq A$. (5) Infra preopen if $A \subseteq Int_i(Cl_i(A))$. (6) Infra preclosed if $Cl_i(Int_i(A)) \subseteq A$. (7) Infra α-open if $A \subseteq Int_i(Cl_i Int_i((A)))$. (8) Infra α-closed if $Cl_i(Int_i(Cl_i(A))) \subseteq A$. (9) Infra b-open if $A \subseteq Int_i(Cl_i(A)) \cup Cl_i(Int_i(A))$. (10) Infra b-closed if $Int_i(Cl_i(A)) \cup Cl_i(Int_i(A)) \subseteq A$. (11) Infra β-open if $A \subseteq Cl_i(Int_i(Cl_i(A)))$. (12) Infra β-closed if $Int_i(Cl_i Int_i((A))) \subseteq A$.

2.9. Definition

A subset K of an infra topological space (X, i_X) is called an infra regular closed (resp. an infra α-closed, an infra semi-closed, an infra preclosed, an infra b-closed and an infra β-closed) if its complements is an infra regular open (resp. an infra α-open, an infra semi-open, an infra preopen, an infra b-open and an infra β- open).

The implication between some types of infra openness is given by Fig. 1.

The family of all infra regular open (resp. infra α-open, infra semi-open, infra preopen, infra b-open and infra β- open) sets in an infra topological space (X, i_X) is denoted by $IRO\,(X, i_X)$ (resp. $I\alpha O(X, i_X)$, $ISO\,(X, i_X)$, $IPO\,(X, i_X)$, $IBO(X, i_X)$ and $I\beta O(X, i_X)$).

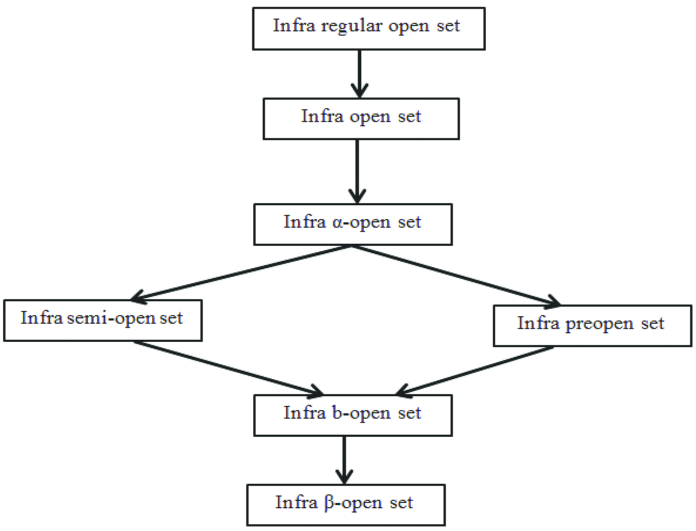

Fig. 1. Comparison between infra open sets.

2.10. Remark

In general, the reverse of the above implication is not true as shown by the following examples.

2.11. Example

(a) Let $X = \{a, b, c\}$ with the following structures:

(i) If $i_X = \{X, \phi, \{b\}, \{c\}\}$. Then $\{a, c\}$ is infra β-open but not infra preopen. Also, it is infra b-open but not infra preopen.

(ii) If $i_X = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}\}$. Then one can deduce that $\{b, c\}$ is an infra α-open set, but it is not infra open set.

(iii) If $i_X = \{X, \phi, \{a\}, \{c\}\}$. Then $\{b, c\}$ is an infra semi-open set, but it is not infra α-open set.

(b) Let $X = \{a, b, c, d\}$ with $i_X = \{X, \phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{b, d\}, \{a, b, c\}\}$. Then one can deduce that $\{a, d\}$ is infra β-open but not infra b-open. Also, $\{c, d\}$ is infra open but not infra regular open.

2.12. Observation

As known in general topological spaces:

(i) The closure of any set is the smallest closed set containing it, while this fact is not satisfied in infra topology.

(ii) The interior of any set is the largest open set contained in it, while this fact is not satisfied in infra topology.

2.13. Example

Let $X = \{a, b, c, d\}$ *with* $i_X = \{\phi, X, \{b\}, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{b, d\}, \{a, b, c\}\}$. *Then:*

(i) If $A = \{b, c, d\}$, then $Int_i(A) = \{b, c, d\}$ which is not infra open.

(ii) If $B = \{a\}$, then $Cl_i(B) = \{a\}$ which is not infra closed.

2.14. Definition

An infra topological space (X, i_X) *is called an infra extremally disconnected if the infra closure of each an infra open subset of* X *is an infra open, or equivalently, if every an infra regular closed subset of* X *is an infra open.*

2.15. Definition

Let (X, i_X) *and* (Y, i_Y) *be infra topological spaces. A mapping* $f: (X, i_X) \longrightarrow (Y, i_Y)$ *is said to be:*

(i) An infra continuous if $f^{-1}(B)$ is an infra open set in X for every an infra open set B in Y.

(ii) An infra α-continuous if $f^{-1}(B)$ is an infra α-open set in X for every an infra open set B in Y.

(iii) An infra semi-continuous if $f^{-1}(B)$ is an infra semi-open set in X for every an infra open set B in Y. (iv) An infra precontinuous if $f^{-1}(B)$ is an infra preopen set in X for every an infra open set B in Y. (v) An infra b-continuous if $f^{-1}(B)$ is an infra b-open set in X for every an infra open set B in Y.

2.16. Proposition

For every an infra topological space (X, i_X)*, we have that:* $IPO(X, i_X) \cup ISO(X, i_X) \subseteq IBO(X, i_X) \subseteq I\beta O(X, i_X)$ *holds but none of these implications can be reversed.*

2.17. Proposition

Let (X, i_X) *be an infra topological space. Then:*

(i) If $A \subseteq X$ *is an infra open and* $B \subseteq X$ *is an infra semi-open (resp. an infra preopen, an infra* β*-open, an infra* b*-open) then* $A \cap B$ *is an infra semi-open (resp, an infra preopen, an infra* β*-open, an infra* b*-open).*

(ii) For every subset $A \subseteq X$, $A \cap Int_i(Cl_i(A))$ is an infra preopen.

(iii) $A \subseteq X$ is an infra b-open, if and only if A is the union of a an infra semi-open set and an infra preopen set.

2.18. Proposition

If (X, i_X) *is an infra extremally disconnected space. Then, the following statements hold:*

(i) Each an infra β*-open set is infra preopen.*

(ii) Each an infra β*-closed set is infra preclosed.*

(iii) Each an infra semi-open set is infra α*-open. (iv) Each an infra semi-closed set is infra* α*-closed.*

Proof. It follows from the fact that if (X, i_X) is infra extremally disconnected, then the notions of infra α-open sets, infra semi-open sets, infra preopen sets and infra β-open sets are equivalent.

2.19. Proposition

For an infra topological space (X, i_X)*, the following properties are equivalent:*

(1) (X, i_X) *is an infra extremally disconnected space.*

(2) $ISO(X, i_X) \subseteq IPO(X, i_X)$. *(3)* $I\beta O(X, i_X) = IPO(X, i_X)$. *(4)* $IBO(X, i_X) = IPO(X, i_X)$

Proof. (1)⇔ (2) and (1)⇔ (3) these are obvious. Clearly, (3)⇔ (4) and (4)⇔ (1) follow immediately from Proposition 2.16.

2.20. Proposition

The intersection of an infra preopen set and an infra α*-open set is infra preopen.*

Proof. Let $A \in \mathrm{IPO}(X, i_X)$ and $B \in \mathrm{I\alpha O}(X, i_X)$, then $A \subseteq Int_i(Cl_i(A))$ and $B \subseteq Int_i(Cl_i(Int_i((B)))$. So $A \cap B \subseteq Int_i(Cl_i(A)) \cap Int_i(Cl_i(Int_i((B))) \subseteq Int_i[Int_i(Cl_i(A)) \cap Cl_i(Int_i(B))] \subseteq Int_i[Cl_i[Cl_i(A) \cap Int_i(B)]] \subseteq Int_i[Cl_i[Cl_i[A \cap B]]] \subseteq Int_i(Cl_i(A \cap B)$. Hence $A \cap B$ is infra preopen.

2.21. Corollary

The union of an infra preclosed set and an infra α*-closed set is an infra preclosed set.*

2.22. Proposition

The intersection of an infra α*-open set and an infra* β*-open set is infra* β*-open.*

Proof. Obvious.

2.23. Corollary

The union of an infra α*-closed set and an infra* β*-closed set is infra* β*-closed.*

2.24. Remark

The arbitrary intersection of infra β*-closed sets is infra* β*-closed but the union of two infra* β*-closed sets may not be an infra* β*-closed set. This is clearly by the following example.*

2.25. Example

Let $X = \{a,b,c,d\}$ with $i_X = \{\phi, X, \{b\}, \{c\}, \{d\}, \{a,c\}, \{c,d\}, \{b,d\}, \{a,b,c\}\}$, the subsets $F = \{b\}$ and $W = \{a,d\}$ are infra β-closed sets, but $F \cup W = \{a,b,d\}$ is not an infra β-closed set.

2.26. Proposition

Each of an infra β-closed set which is an infra semi-closed set is an infra semi-open.

Proof. Let A be an infra β-closed set and infra semi-closed. Then, $A \subseteq Cl_i(Int_i(Cl_i(A)))$ and $Int_i(Cl_i(A)) \subseteq A$. Therefore, $Int_i(Cl_i(A)) \subseteq Int_i(A)$ and so, $Cl_i(Int_i(Cl_i(A))) \subseteq Cl_i(Int_i(A))$. Hence, $A \subseteq Cl_i(Int_i(Cl_i(A))) \subseteq Cl_i(Int_i(A))$. This means that A is an infra semi-open.

2.27. Proposition

A subset F of an infra topological space (X, i_X) is an infra β-closed if and only if $Cl_i(X \backslash Cl_i(Int_i(F))) \backslash (X \backslash Cl_i(F)) \supseteq Cl_i(F) \backslash F$.

Proof. $Cl_i(X \backslash (Cl_i(Int_i(F))) \backslash (X \backslash Cl_i(F)) \supseteq Cl_i(F) \backslash F$ if and only if $(X \backslash Int_i(Cl_i(Int_i(F)))) \backslash (X \backslash Cl_i(F)) \supseteq Cl_i(F) \backslash F$ if and only if $(X \backslash Int_i(Cl_i(Int_i(F))) \cap Cl_i(F) \supseteq Cl_i(F) \backslash F$ if and only if $(X \cap Cl_i(F)) \backslash (Int_i(Cl_i(Int_i(F)))) \cap Cl_i(F))) \supseteq Cl_i(F) \backslash F$ if and only if $Cl_i(F) \backslash (Int_i(Cl_i(Int_i(F))) \supseteq Cl_i(F) \backslash F$ if and only if $F \supseteq Int_i(Cl_i(Int_i(F)))$ if and only if F is is an infra β-closed.

2.28. Proposition

Let F be a subset of of an infra topological space (X, i_X). If F is an infra β-closed set and an infra semi-open set, then it an infra semi-closed set.

Proof. Since F is an infra β-closed set and an infra semi-open set then $X \backslash F$ is an infra β-open and an infra semi-closed and so by Proposition 2.26. $X \backslash F$ is an infra semi-open. Therefore, F is an infra semi-closed.

2.29. Proposition

Each an infra β-open set and an infra α-closed set is an infra regular closed.

Proof. Let $A \subseteq X$ be an infra β-open set and an infra α-closed set. Then $A \supseteq Cl_i(Int_i(Int_i(F)))$ and $Cl_i(Int_i(Int_i(F))) \supseteq A$, which implies that $Cl_i(Int_i(Int_i(F))) \supseteq A \supseteq Cl_i(Int_i(Int_i(F)))$. So, $A = Cl_i(Int_i(Int_i(F)))$. This means that A is an infra closed, and so it is an infra regular closed.

2.30. Corollary

Each an infra β-closed set and an infra α-open set is infra regular open.

2.31. Lemma

Let (X, i_X) be an infra topological space and $A, B \subseteq X$. Then the following properties hold:

(i) $Int_i(A) \cap Int_i(B) = Int_i(A \cap B)$,

(ii) $Cl_i(A) \cup Cl_i(B) = Cl_i(A \cup B)$.

2.32. Proposition

Let (X, i_X) be an infra topological space. Then for $A \subseteq X$, $A \cup Int_i(Cl_i(A))$ is an infra semi-closed.

Proof. From Lemma 2.31, $Int_i(Cl_i(A \cup Int_i(Cl_i(A)))) = Int_i(Cl_i(A) \cup Cl_i(Int_i(Cl_i(A)))) = Int_i(Cl_i(A)) \subseteq A \cup Int_i(Cl_i(A))$. So, $A \cup Int_i(Cl_i(A))$ is an infra semi-closed.

3. Some Classes of Infra Continuity

3.1. Definition

Let (X, i_X) and (Y, i_Y) be infra topological spaces. The mapping $f: (X, i_X) \longrightarrow (Y, i_Y)$ is said to be infra β-continuous or (an infra semi-pre-continuous) if $f^{-1}(A)$ is an infra β-open set in X for every an infra open set A in Y.

The relations between the above types of infra near continuous functions is clearly by the diagram in Fig. 2.

Now, we show that, none of these implications is reversible as shown by the following examples.

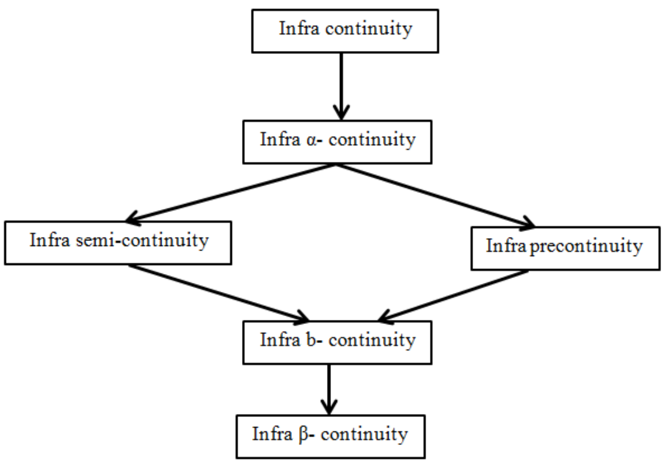

Fig. 2. Comparison between types of an infra continuity

3.2. Example

Let $X = \{a, b, c, d\}$ with $i_X = \{\phi, X, \{a, d\}\}$ and $Y = \{x, y\}$ with $i_Y = \{\phi, Y, \{x\}, \{y, w\}\}$. Define $f: (X, i_X) \longrightarrow (Y, i_Y)$ as; $f(a) = y$, $f(b) = y$, $f(c) = z$ and $f(d) = w$. Then f is an infra α-continuous but not an infra continuous.

3.3. Example

Let $X = \{a, b, c, d\}$ with $i_X = \{\phi, X, \{a, d\}\}$ and $Y = \{x, y, z, w\}$ with $i_Y = \{\phi, Y, \{x\}, \{y, w\}\}$. Define $f: (X, i_X) \longrightarrow (Y, i_Y)$ as; $f(a) = y, f(b) = y, f(c) = z$ and $f(d) = w$. Then f is an infra semi-continuous but not an infra α-continuous. Also, f is an infra b-continuous but not an infra precontinuous.

3.4. Example

Let (X, i_X) and (Y, i_Y) be be infra topological spaces defined as in Example 3.3 and let $g: (X, i_X) \longrightarrow (Y, i_Y)$ be defined as follows: $f(a) = w, f(b) = y, f(c) = z, f(d) = w$. Then g is an infra α-continuous but not infra b-continuous.

3.5. Example

Let (X, i_X) and (Y, i_Y) be be infra topological spaces defined as in Example 3.3 and let $h: (X, i_X) \longrightarrow (Y, i_Y)$ be defined as follows: $h(a) = y$, $h(b) = x$, $h(c) = z$, $h(d) = w$. Then h is an infra b-continuous but not infra semi-continuous.

3.6. Example

Let (X, i_X) and (Y, i_Y) be be infra topological spaces defined as in Example 3.3 and let $f: (X, i_X) \longrightarrow (Y, i_Y)$ be a mapping defined as follows: $h(a) = w$, $f(b) = x$, $f(c) = w$, $f(d) = x$. Then f is an infra precontinuous but not infra α-continuous.

3.7. Theorem

Let (X, i_X) and (Y, i_Y) be infra topological spaces, and let $f: (X, i_X) \longrightarrow (Y, i_Y)$ be a mapping. Then, the following statements are equivalent:

(i) f is an infra β-continuous.

(ii) The inverse image of every infra closed set F in Y is an infra β-closed in X. (iii) $f(\beta Cl_i(A)) \subseteq Cl_i(f(A))$, for every subset A of X. (iv) $\beta Cl_i(f^{-1}(F)) \subseteq f^{-1}(Cl_i(F))$, for every subset F of Y. (v) $f^{-1}(Int_i(F)) \subseteq \beta Int_i(f^{-1}(F))$, for every subset F of Y.

Proof. **(i)⟹ (ii)**: Let f be an infra β-continuous and F be an infra closed set in Y. That is $Y \backslash F$ is an infra open in Y. Since f is an infra β-continuous mapping. Then $f^{-1}(Y \backslash F)$ is an infra β-open in X. Then $f^{-1}(Y \backslash F) = X \backslash f^{-1}(F)$ which means that, $f^{-1}(F)$ is is an infra β-closed in X. (ii)⟹ (i): Let G be an infra open set in Y . Then, $f^{-1}(Y \backslash G)$ is an infra β-closed in X. Then $f^{-1}(G)$ is an infra β-open in X. Therefore, f is an infra β-continuous mapping.

(i)⟹ (iii): Let f be an infra β-continuous and let $A \subseteq X$. Since f is nano β-continuous and $Cl_i f(A)$ is an infra closed in Y, $f^{-1}(Cl_i f(A))$ is an infra β-closed in X. Since $f(A) \subseteq Cl_i f(A)$, $f^{-1}(f(A)) \subseteq f^{-1}(Cl_i f(A))$, then $\beta Cl_i(A) \subseteq \beta Cl_i[f^{-1}(Cl_i f(A))] = f^{-1}(Cl_i f(A))$. Thus $\beta Cl_i(A) \subseteq f^{-1}(Cl_i f(A))$. Therefore, $f(\beta Cl_i(f(A)) \subseteq Cl_i(f(A))$ for every subset A of X.

(iii)⟹ (i): Let $f(\beta Cl_i(A) \subseteq Cl_i(f(A))$ for every subset A of X. Let F be an infra closed in Y , then $f(\beta Cl_i(F) \subseteq Cl_i(f(f^{-1}(F))) = Cl_i(F) = F$ that is $f(\beta Cl_i(f^{-1}(F)) \subseteq F$. Thus $\beta Cl_i(f^{-1}(F)) \subseteq f^{-1}(F)$, but $f^{-1}(F) \subseteq \beta Cl_i(f^{-1}(F))$. Hence $\beta Cl_i(f^{-1}(F)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is an infra β-closed in X for every infra closed set F in Y. That is f is an infra β-continuous.

(i)⟹ (iv): Let f be an infra β-continuous and let $F \subseteq Y$, then $Cl_i(F)$ is an infra closed in Y and hence $f^{-1}(Cl_i(F))$ is an infra β-closed in X. Therefore, $\beta Cl_i[f^{-1}(Cl_i(F))] = f^{-1}(Cl_i(F))$. Since $F \subseteq \beta Cl_i(F)$, $f^{-1}(F) \subseteq f^{-1}(Cl_i(F))$. Then $\beta Cl_i(f^{-1}(F)) \subseteq \beta Cl_i(f^{-1}(Cl_i(F))) = f^{-1}(Cl_i(F))$. Thus $\beta Cl_i(f^{-1}(F)) \subseteq f^{-1}(Cl_i(F))$.

(iv)⟹ (i): Let $\beta Cl_i(f^{-1}(F)) \subseteq f^{-1}(Cl_i(F))$ for every subset F of Y . If F is an infra closed in Y, then $Cl_i(F) = F$. By assumption, $\beta Cl_i(f^{-1}(F)) \subseteq f^{-1}(Cl_i(F)) = f^{-1}(F)$. But $f^{-1}(F) \subseteq \beta Cl_i(f^{-1}(F))$. Therefore, $\beta Cl_i(f^{-1}(F)) = f^{-1}(F)$. That is, $f^{-1}(F)$ is an infra β-closed in X for every infra closed in Y. Therefore f is an infra β-continuous.

(i)⟹ (v): Let f be an infra β-continuous and let $F \subseteq Y$, then $Int_i(F)$ an infra closed in Y and hence $f^{-1}(Int_i(F))$ is an infra β-open in X. Therefore $\beta Int_i[f^{-1}(Int_i(F))] = f^{-1}(Int_i(F))$. Also, $Int_i(F) \subseteq F$ implies that $f^{-1}(Int_i(F)) \subseteq f^{-1}(F)$. Therefore, $\beta Int_i(f^{-1}(Int_i(F))) \subseteq \beta Int_i(f^{-1}(F))$. That is, $f^{-1}(Int_i(F)) \subseteq \beta Int_i(f^{-1}(F))$.

(v)⟹ (i): Let $f^{-1}(Int_i(F)) \subseteq \beta Int_i(f^{-1}(F))$ for every $F \subseteq Y$. If F is an infra open in Y, then $Int_i(F) = F$. By assumption, $f^{-1}(Int_i(F)) \subseteq \beta Int_i(f^{-1}(F))$. Thus $f^{-1}(F)) \subseteq \beta Int_i(f^{-1}(F))$. But $\beta Int_i(f^{-1}(F)) \subseteq f^{-1}(F)$. Therefore, $f^{-1}(F) = \beta Int_i(f^{-1}(F))$. That is, $f^{-1}(F)$ is an infra β-closed in X for every infra open set F in Y. Therefore, f is an infra β-continuous.

3.8. Remark

If $f: (X, i_X) \longrightarrow (Y, i_Y)$ is an infra β-continuous where (X, i_X) and (Y, i_Y) be infra topological spaces. Then $f(\beta Cl_i(A))$ is not necessarily equal to $Cl_i(f(A))$. This is clearly by the following example.

3.9. Example

Let $X = \{a, b, c, d, e\}$ with $i_X = \{\phi, X, \{a, b, c\}\}$ and $Y = \{u, v, x, y, z\}$ with $i_Y = \{\phi, Y, \{u, v, z\}\}$. Define $f: (X, i_X) \longrightarrow (Y, i_Y)$ as; $f(a) = x$, $f(b) = x$, $f(c) = u$, $f(d) = v$ and $f(e) = y$. Clearly, f is an infra β-continuous. Let $A = \{a, b, c\} \subseteq Y$. Then $f(\beta Cl_i(A)) = f(\{a, b, c, d, e\}) = \{u, v, x, y\}$. But, $Cl_i(f(A)) =$

$Cl_i(\{x,u\}) = Y$. *Thus* $f(\beta Cl_i(A)) \neq Cl_i(f(A))$.

3.10. REMARK

In Theorem 3.7 equality of the statements (iv) and (v) does not hold in general as shown by the following example.

3.11. EXAMPLE

Let $X = \{a,b,c,d\}$ *with* $i_X = \{\phi, X, \{c\}, \{a,d\}, \{a,c,d\}\}$ *and* $Y = \{x,y,z,w\}$ *with* $i_Y = \{\phi, Y, \{x,w\}\}$. *Define* $f: (X, i_X) \longrightarrow (Y, i_Y)$ *as;* $f(a) = x$, $f(b) = y$, $f(c) = z$ *and* $f(d) = w$. *Then* f *is an infra* β*-continuous. (i) Let* $F = \{z,w\} \subseteq Y$. *Then* $f^{-1}(Cl_i(F)) = f^{-1}(Y) = X$ *and* $\beta Cl_i(f^{-1}(F)) = \beta Cl_i(\{c,d\}) = \{b,c,d\}$. *Therefore,* $\beta Cl_i(f^{-1}(F)) \neq f^{-1}(Cl_i(F))$.

(ii) Let $F = \{x,y\} \subseteq Y$. Then $f^{-1}(Int_i(\{x,y\})) = f^{-1}(\{x,y\}) = \{a,b\}$ and $\beta Int_i(f^{-1}(F)) = \beta Int_i(\{x,y\}) = \beta Int_i(\{a,b\}) = \{a\}$. Therefore, $f^{-1}(Int_i(F)) \neq \beta Int_i(f^{-1}(F))$.

4. CONCLUSIONS AND FUTURE WORK

In this paper the properties of some infra near open sets and some types of an infra continuity are discussed. A comparison between near infra open sets and some other types of near open sets is investigated. Also, A comparison between near infra open sets and some other types of near open sets is obtained. So, we introduce some examples to satisfy a given relationships. Thus, it is advantageous to use an infra topology in real life situations.

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